

MU-TONICS: IN SEARCH OF MUTABLE TECTONICS

An investigation into natural systems with efficient packing & stacking strategies leading to a design methodology of an adaptive (mutable) system in architecture.

A Thesis
Presented to
The Academic Faculty

By

Lorraine Grace G. Ong

In Partial Fulfillment
Of the Requirements for the Degree
Master of Science in Architecture

Georgia Institute of Technology

May, 2007

MU-TONICS: IN SEARCH OF MUTABLE TECTONICS

An investigation into natural systems with efficient packing & stacking strategies leading to a design methodology of an adaptive (mutable) system in architecture.

Prof. Lars Spuybroek, Advisor
College of Architecture
Georgia Institute of Technology

Dr. Athanassios Economou
College of Architecture
Georgia Institute of Technology

Dr. T. Russell Gentry
College of Architecture
Georgia Institute of Technology

Date Approved
April 9, 2007

ACKNOWLEDGEMENTS

I would like to thank my advisor, Prof. Lars Spuybroek and the rest of my committee members, Dr. Athanassios Economou and Dr. T. Russell Gentry for providing me both with the conceptual and technical knowledge to engage in this explorative undertaking.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS.....	III
LIST OF FIGURES.....	V
SUMMARY	VII
1 INTRODUCTION.....	1
2 FREI OTTO – ARCHITECTURE OF THE MINIMAL	5
2.1 NATURAL CONSTRUCTIONS	5
2.2 ANALOGUE COMPUTATION.....	7
3 OBSERVATIONS ON NATURAL PACKING / STACKING SYSTEMS	11
3.1 ANIMATE - BONE TISSUE.....	12
3.2 INANIMATE - FOAM / SOAP BUBBLES.....	14
3.2.1 <i>Crystallized Structures / Membranes of Foam (Solid)</i>	15
3.2.2 <i>Trapped Air in Soap Bubbles (Void)</i>	16
3.3 GEOMETRIC - SPHERE PACKING	18
4 MACHINING EMERGENCE	20
4.1 PACKING VS. STACKING.....	20
4.2 METHODOLOGY	21
4.3 ANALOGUE EXPERIMENTS	22
4.3.1 <i>Homogeneous Packing</i>	22
4.3.2 <i>Homogeneous Stacked</i>	32
4.3.3 <i>Heterogeneous / Mixed</i>	37
4.3.4 <i>Summary of Findings</i>	42
4.4 GRID / PATTERN LAYERING	44
4.4.1 <i>Vertical Relationships</i>	44
4.4.2 <i>Vertical Transformations</i>	50
4.4.3 <i>Horizontal Relationships</i>	52
4.4.4 <i>Horizontal Transformations</i>	56
4.4.5 <i>Conclusion</i>	58
REFERENCES	59

LIST OF FIGURES

Figure 1-1: a) Beehive; b) Microscopic Image of Foam.....	3
Figure 1-2: Manifold Project, Emergence & Design Group AA.....	4
Figure 2-1: a) Branching Studies; b) Textile Membranes for The German Pavilion.....	5
Figure 2-2: Soap Bubbles Representing Crystal Structures of Matter.....	7
Figure 2-3: 2D Analogue Experiments on Direct Path Systems	8
Figure 2-4: 3D Thread Model As Representations of Biological Structures	9
Figure 3-1: Bone Structure Example – Beak of A Black Stork.....	11
Figure 3-2: Detail of Bone Tissue	12
Figure 3-3: Ossification Process – Bone Growth	13
Figure 3-4: Foam Formation Process	15
Figure 3-5: Open Cell Structured Foam & Closed Cell Structured Foam.....	16
Figure 3-6: Forces Acting Upon a Surface Film Between 4 Soap Bubbles	17
Figure 3-7: Sphere Packing Techniques.....	19
Figure 4-1: Experiment 1.1 _ Face-Centered Cubic Packing	22
Figure 4-2: Experiment 1.1 _ Digital Process Illustrating the Reconstruction of Members	23
Figure 4-3: Experiment 1.1 _ 3-Sided Members As Connection Between Voids	24
Figure 4-4: Experiment 1.2 _ Hexagonal Packing	25
Figure 4-5: Experiment 1.2 _ Digital Process Illustrating the Reconstruction of Members	26
Figure 4-6: Experiment 1.2 _ 4-Sided Member As Connection Between Voids.....	27
Figure 4-7: Experiment 1.3 _ Hexagonal Packing - Dense.....	28
Figure 4-8: Experiment 1.3 _ Digital Process Illustrating the Reconstruction of Members	29
Figure 4-9: Experiment 1.3 _ 4-Sided Dense Member	30
Figure 4-10: Summary of Members from Packing Strategies.....	31
Figure 4-11: Experiment 2.1 _ Triangular Grid Pattern in a Straight Stack	32
Figure 4-12: Experiment 2.1 _ Digital Process Illustrating the Reconstruction of Members	32
Figure 4-13: Experiment 2.1 _ 3-Sided Members As Connection Between Voids	33

Figure 4-14: Experiment 2.2 _ Square Grid Patterns in a Straight Stack	34
Figure 4-15: Experiment 2.2 _ Digital Process Illustrating the Reconstruction of Members	34
Figure 4-16: Experiment 2.2 _ 4-Sided Members As Connection Between Voids	35
Figure 4-17: Summary of Members from Stacking Strategies.....	36
Figure 4-18: Experiment 3.1 _ Digital Process Illustrating the Reconstruction of Members	37
Figure 4-19: Experiment 3.1 _ Random Packing & Stacking Creates Variable Member Patterns	38
Figure 4-20: Experiment 3.1 _ 3-Sided Member Meets a 4-Sided Member	38
Figure 4-21: Experiment 3.2 _ Packing + Vertical Anomalous Object.....	39
Figure 4-22: Experiment 3.3 _ Packing + Horizontal Anomalous Object	40
Figure 4-23: Experiment 3.2 _ Digital Process Illustrating the Reconstruction of Members	41
Figure 4-24: Experiment 3.2 & 3.3 _ Flat Face of Member Creates Surface Boundary of Aperture.....	41
Figure 4-25: Summary of Findings from Analogue Experiments (Packed / Stacked)	42
Figure 4-26: Vertical Relationship _ Face-Centered Packing.....	44
Figure 4-27: Vertical Relationship _ Hexagonal Packing.....	45
Figure 4-28: Vertical Relationship _ Triangular Grid Pattern Stacked	46
Figure 4-29: Vertical Relationship _ Square Grid Pattern Stacked	47
Figure 4-30: Vertical Relationship _ Triangular Grid + Square Grid	48
Figure 4-31: Vertical Relationship _ Square Grid + Triangular Grid.....	49
Figure 4-32: Matrix of Vertical Transformations.....	50
Figure 4-33: Example of Vertical Transformation of Members, Redrawn Considering Loads	51
Figure 4-34: Horizontal Relationship _ Face-Centered Packing + Hexagonal Packing + Dense..	52
Figure 4-35: Horizontal Relationship _ Triangular Grid Stack + Square Grid Stack + Dense	53
Figure 4-36: Horizontal Relationship _ Face-Centered Packing + Triangular Grid Stack + Hybrid.....	54
Figure 4-37: Horizontal Relationship _ Hexagonal Packing + Square Grid Stack + Hybrid	55
Figure 4-38: Matrix of Planar Connectivity Exhibiting Possible Horizontal Transformations.....	57
Figure 4-39: Detail of 4-Sided Member Distorted due to Vertical Relationship	57
Figure 4-40: Axonometric View of Structure & Surface Fills.....	58
Figure 4-41: Perspective Rendering of Structure + Surface Fill System	59

SUMMARY

In search of mutable tectonics is a research investigation linking principles found in natural systems, investigated by various fields in biology, physics, and mathematics, in the creation of a design methodology in Architecture. Specifically the report looks into natural system with packing and stacking strategies like bone formation, foam or soap bubbles, and sphere packing. Rules and physical observations of the natural are carried forward in the development of a topological language, through digital investigations, which define relationships between variations in spatial configurations and structural members. What we hope to achieve here is that by studying natural systems already realized in the natural world a more adaptive system of design between form, structure and space is immediately established; resulting in the discovery of emergent spaces which intrinsically conveys an emergent structural system and vice versa. The outcome is the creation of an adaptive networked process in the design formulation in Architecture.

1 Introduction

(mu-ta-ble): mutare – to change, capable of change or of being changed

(tec-ton-ics): tekton – builder

The idea of mutability in architecture seems far removed from Architecture which is deeply rooted in the physical world. Buildings do not reconfigure and readjust themselves to meet certain parameters. Instead what is of importance, in this study, is the concept of adaptation in the formation of mutations. The key word here being adaptation implies not only the potential for change but more importantly is a reaction to certain causalities. This form-finding process of mutation through adaptation is reciprocal in the practice of Architecture. Seeing as Architecture defines a form given certain factors like site, space, environmental factors, etc... How do all these varying causalities define or direct the formal construction in Architecture?

Adaptation or mutation is more clearly seen in the natural world. It is in fact a key principle in the theory of evolution (Darwin). Nature is the most efficient builder. It recreates, reconfigures, organisms to rework the manner in which they deal with environmental changes; a self-organizing process. This self-organizing process in the creation of form has, in the last 10 years, been a large area of interest for biologist, mathematicians, physicist and architects. The latter of which defines these concerns in research on morphogenetic strategies and the construction of emergence.

Emergence defined In discussion of complex, adaptive systems:

“The movement from low-level rules to higher-level sophistication is what we call emergence, and ... a higher-level pattern arising out of parallel complex interactions between local agents.”¹

This method of research in constructing emergence provides the opportunity to explore different natural systems. Nature itself, the master builder, provides a framework or a model to begin our investigations. Biological forms are inherently composed of a multitude of simpler forms which reconfigure to create a complex whole. They adapt, heal, themselves against environmental factors or forces; an intrinsically complex process in the production of form and complex behavior.

Can we learn something from nature's efficient processes of evolution or mutation and apply this into the adaptive process of architectural design?

Given the broad formulation of possibilities or studies dealing with natural systems, this paper will only look at natural systems exemplifying packing and stacking strategies. Packing and stacking strategies can be found from the smallest particles in nature, like in cellular aggregation or formulation in bones, to physically observed phenomena's like in the making of beehives.

¹ Steven Johnson, Emergence: The Connected Lives of Ants, Brains, Cities, and Software. (New York: Scribner, 2001)

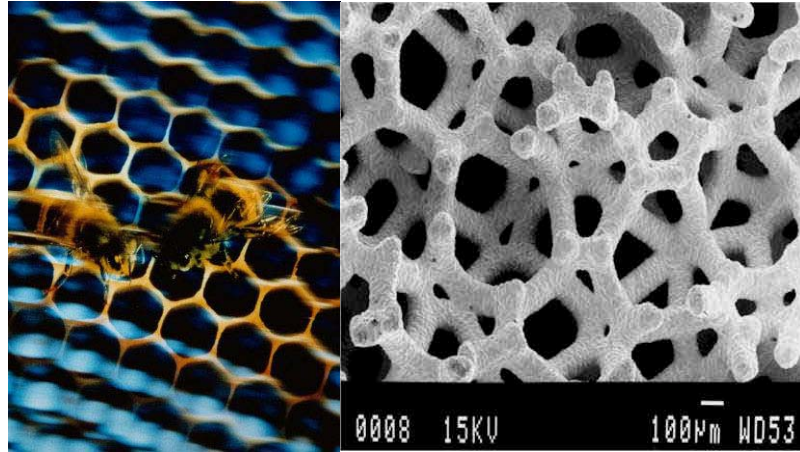


Figure 1-1: a) Beehive; b) Microscopic Image of Foam

Historically speaking, looking towards nature as an inspiration for design is not a new concept. Antonio Gaudi conceived of the Sagrada Familia by means of a complex analogue system of catenary curves. Frei Otto systematically devised analogue experiments to study minimal surfaces and applied these concepts in his complex tent structures. Today these analogue models recreating natural systems are coupled with digital means (computation). The computer has provided a powerful tool in defining these complex relationships which exists in nature's self-organizing process. Digital computation has provided the means to examine these relationships and in the creation of variations from these systems – digital morphogenesis.²

An exemplary precedent is the manifold project developed by the emergent and design group in the London based Architectural Association. The manifold project takes the simple operations inherent in the honeycomb system found in beehives and explored its generative possibilities for form. The group systematically did material and fabrication research culminating in a full-scale installation demonstrating the systems formal capabilities.

² Branko Kolarevic, Architecture in the Digital Age: Design and Manufacturing. (New York: Spon Press. 2003) ch 2.

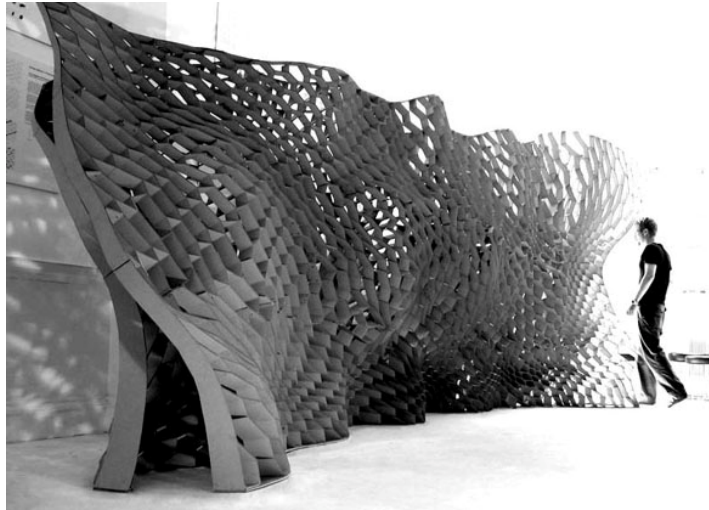


Figure 1-2: Manifold Project, Emergence & Design Group AA

Clearly these concepts of emergence and digital morphologies have richly broadened physical possibilities in architecture beyond its static notion of form. The search for the natural in Architecture, through research of natural systems, will produce material innovations, a more integrated network of spatial and formal concerns, and an overall more adaptive process of creation.

2 Frei Otto – Architecture of the Minimal

Frei Otto, an Architect and Engineer, led the way in developing minimal structures through cross-disciplinary research in the fields of biology and architecture. In 1964, he established the Institute for Lightweight Structures in the University of Stuttgart. This organization has been responsible for numerous constructions of complex non-linear geometric forms.

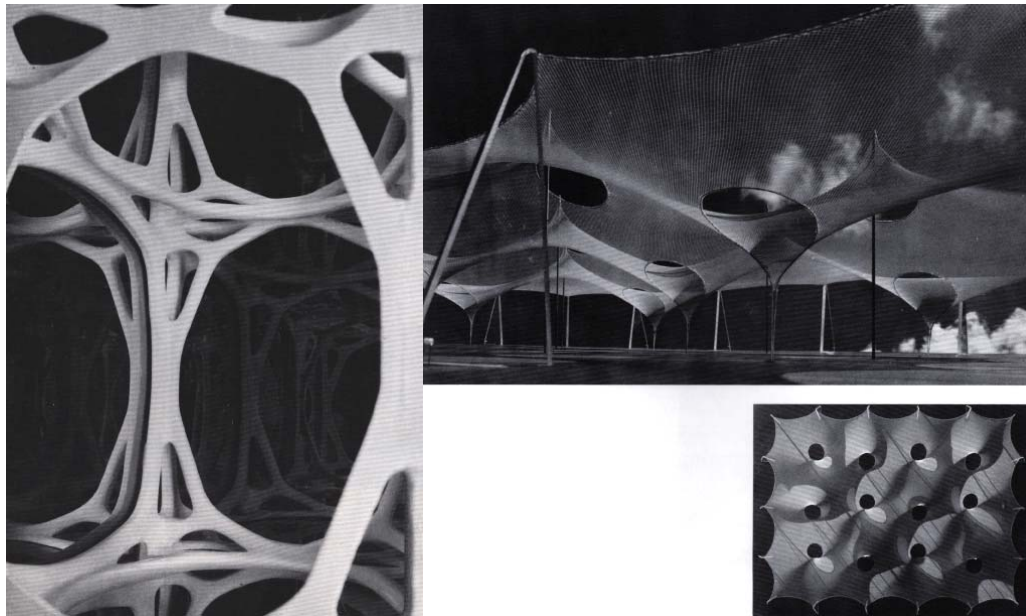


Figure 2-1: a) Branching Studies; b) Textile Membranes for The German Pavilion

2.1 *Natural Constructions*

“...natural objects are natural constructions. They come into being as a result of self-formation processes.”³

³ Frei Otto and Bodo Rasch. Finding Form: Towards an Architecture of the Minimal. (Germany: Axel Menges. 1995)

“Inanimate nature is not disintegrating, as was once postulated, to the point of chaotic unrecognizability. It is in a state of permanent transformation, something that is briefly chaotic constantly rearranges itself into new objects & constructions. It forms the new shapes of natural constructions.”⁴

In 1991, Frei Otto delivered a lecture on natural constructions. In his lecture he does not distinguish nature as being solely biological or artificial / man-made. Instead natural objects and man-made objects are intrinsically similar since both are a product of a process of construction. As a product of construction, we come to recognize that material or physical objects are composed of a multitude of parts; whether we are looking at an atomic view of an organic matter (i.e. microscopic image of metals) or an impressionist painting by Georges Seurat. At the same time, this process of construction (for biological forms refers to the natural process of evolution) materializes variable instances of realizations through the random composition of parts.

Inherently, we acknowledge that the process of evolution for biological forms and its form-finding technique is comparable to the creation of artificial forms; and perhaps in a larger context even the creation of Architectural forms. Simultaneously, recognizing the irregularity between biological and artificial forms in their process of making; the former being more efficient in the creation of instances which adapt to certain utilization factors through its ability for self-organization. For this reason there is a need for research into biological processes in search for techniques which may be adapted to the design of more natural buildings.

“...Our times demand lighter, more energy-saving, more mobile and more adaptable, in short more natural buildings... The search for the natural in Architecture does not restrict the possibilities, it extends them. It creates the condition that our buildings can for once be less unnatural than they have been previously.”⁵

⁴ Otto, Finding Form 10.

⁵ Otto, Finding Form 17.

2.2 Analogue Computation

Frei Otto uses simple analogue models to represent biological forms and to simulate natural processes or phenomenon. He calls this the reverse path method which does not literally make a copy of nature (i.e. like the germination of cells in research laboratories) but is "... made comprehensible through technical developments."⁶ Figure 2-2 demonstrates this method by the use of soap bubbles to represent crystal structures of matter. The close packing of soap bubbles which uses principles of surface tension to achieve equilibrium reproduces the process of creation of crystalline aggregation in metals. The scientific or technical understanding of the formation of soap bubbles makes crystal aggregation comprehensible.

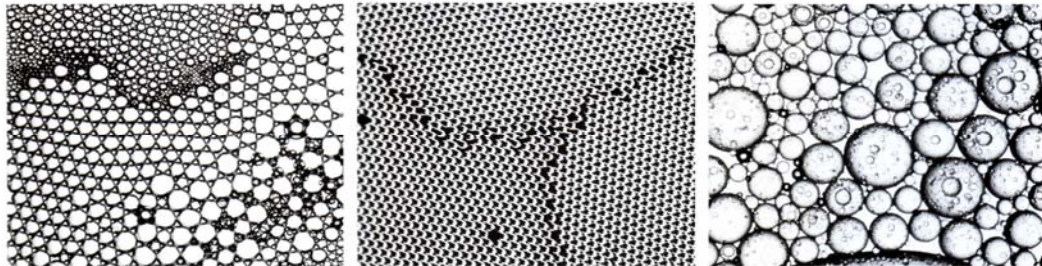


Figure 2-2: Soap Bubbles Representing Crystal Structures of Matter

(l-r) a) bubbles covering a surface; b) surface covered by bubbles explain crystal construction; c) speck of fat of various sizes demonstrate surface occupation

One of his more famous experiments was on his research on minimal path systems. By means of a 2D thread model, some pins and soap water, Frei Otto was able to create a machine which generates variable outcomes of optimized path systems. This

⁶ Otto, *Finding Form* 45.

self-organization process creates instances or mutations of the direct path; which at the same time optimizes possibilities due to the nature of soap film.

“Soap films always shrink to the smallest possible surface area: the so-called minimal surface configuration.”⁷ This means that soap films naturally determine the most optimum spatial configuration given a set number of points (control points).

The research on minimal path systems shed light on minimal requirements needed to bridge between an idealized direct path and a more realistic solution; which in its largest context may be equated with city planning and circulation techniques. Figure 2-3 illustrates this 2D experiment showing a direct path system, an optimized minimal path system and some details of branching between 24 points, 6 points, and 4 points.

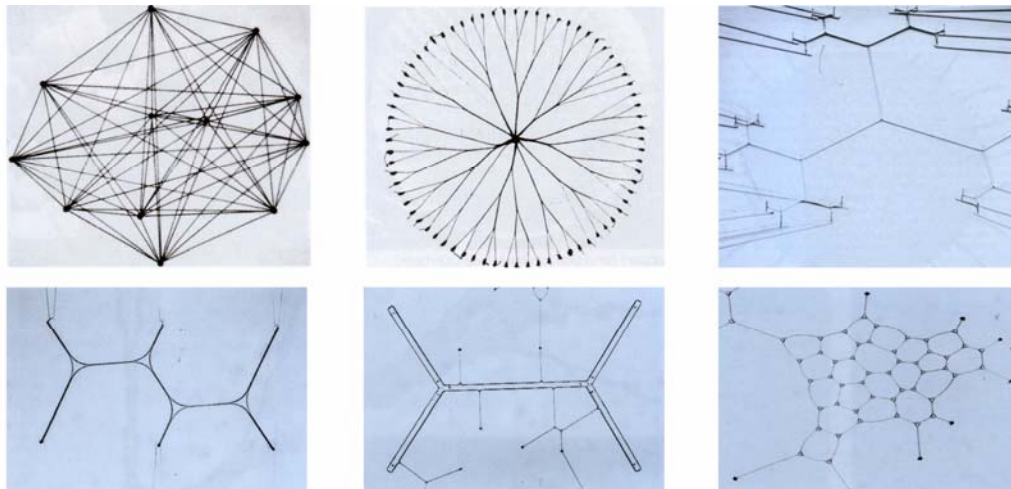


Figure 2-3: 2D Analogue Experiments on Direct Path Systems

(l-r) a) direct path system (minimal energy consumption, large path lengths); b) energy optimization by limited detours (self-formation in damp thread network); c) minimal path system (shortest possible overall path length, between 24 points); d) minimal path system (between 6 points); e) minimal path system (at a angle of 90degrees of fixed elements between 4 points); f) minimal path system with closed meshes

In biological forms the problem of minimal paths can be observed in cellular compositions, crystallized atomic formations in matter, and in foam; taking the

⁷ Otto, Finding Form 50.

understanding of 2D optimization patterns into 3-dimensional forms. Figure 2-4 takes the 2D thread model experiment and conceptualizes the problem of minimal paths in 3D space. The 3D thread model can be used to generate variable solutions to the issue of space packing which is easily observed in the natural creation of cellular structures in bones.

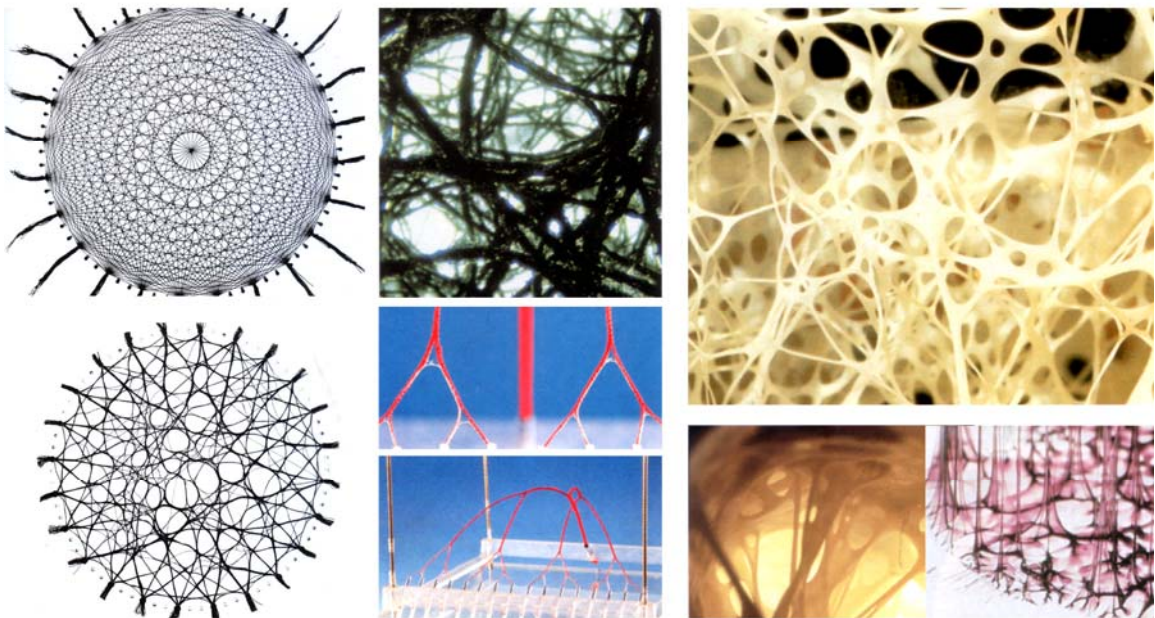


Figure 2-4: 3D Thread Model As Representations of Biological Structures

(l-r per column) a) 2D thread model; b) 3D thread model; c) bone structure created by self-formation

This 3D thread model experiment on minimal path systems is comparable to the architectural problems of space planning or optimization and the design of an optimum structural technique. Essentially these experiments deal with issue of generating solids (structural configurations) or configurations created in the optimization of bridging voids (spatial configurations).

➤ Brief Description of Minimal Path System Experiment

“The so-called minimal path device can reliably determine a minimal path system between any number of points arranged in any number of ways in a matter of a few seconds. The key feature of the device is a sheet of glass placed precisely horizontally over a basin of water. It is sprayed with water from below. The water has a few drops of soap mixed with it. The sheet of glass is touched with fine adjustable needles from below. If the water level is slowly lowered, films of soap form between the sheet of glass, the surface of the water, and the needles. As the distance of the surface of the water to the underside of the glass is constant, the contact lines of the soap films on the underside of the glass plate have minimal overall length.”⁸

⁸ Otto, Finding Form 70.

3 Observations on Natural Packing / Stacking Systems

Since the earliest man-made innovations, man has looked towards nature or biological systems for inspiration. To this day when we think of the concept of flight we picture Leonardo da Vinci's sketches of wings mimicking the flight of birds.

This section presents scientific research or discoveries on biological forms exhibiting natural packing / stacking abilities. We will first look at the structure of bones and how they are formed, which we noted in the previous section as models which can be investigated through Frei Otto's 3-dimensional thread model experiment. This will be followed by research on foam / soap bubbles which present a more abstract conceptualization of packing. And finally a geometric presentation of sphere packing techniques presents a more mathematical model in understanding packing / stacking systems.

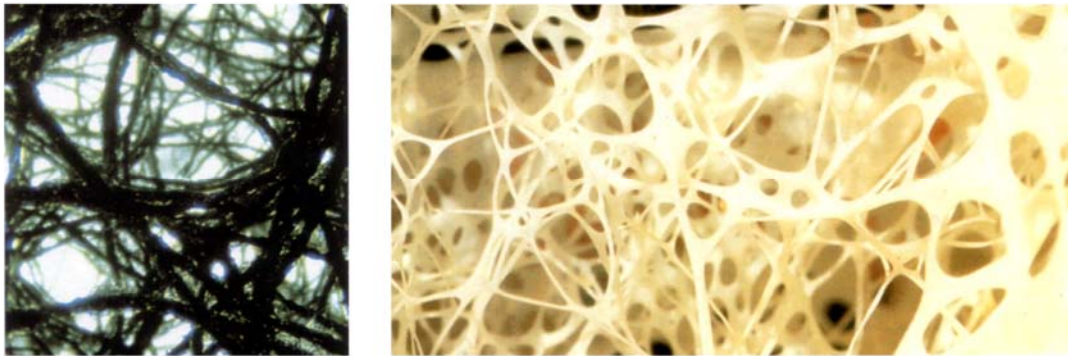


Figure 3-1: Bone Structure Example – Beak of A Black Stork
(l-r) a) 3D thread model experiment; b) bone structure (beak of the black stork)

3.1 Animate - Bone Tissue

Bone tissue or osseous tissue is the major structural and supportive tissue found in bones. This rigid part of the bone is classified into two types: compact and spongy. Compact bone tissue forms the extremely hard exterior of the bone while spongy bone tissue line the inner cavities of bones; having less density and lower strength than compact bone but covering a larger surface area. (See Figure 3.2 for illustrations)

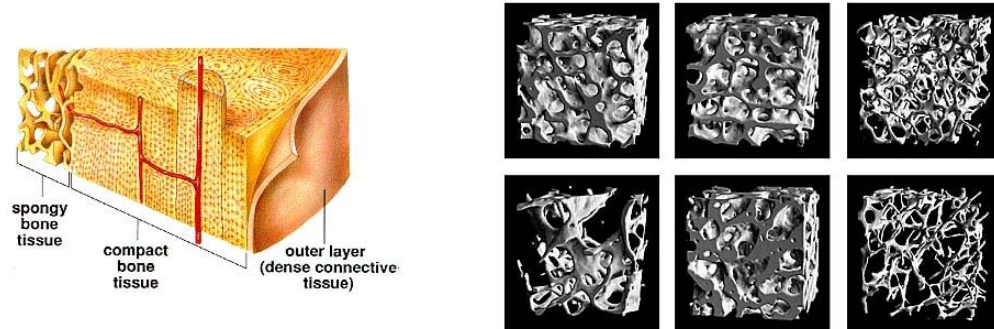


Figure 3-2: Detail of Bone Tissue

(l-r) a) cross-section of bone tissue; b) bone tissue degradation

Bones are formed through the process of endochondral ossification. Endochondral ossification is simply the replacement of hyaline cartilage (cartilage tissue) with bone tissue (either compact or spongy).

The hyaline cartilage consists of cells lying in groups of two or more in a granular or almost homogeneous matrix. During the earliest stages of bone formation the cells (in hyaline cartilage tissues) aggregate to form a more compact grouping of cells. This condensation of cells eventually becomes a calcified cartilaginous matrix; which is the basic template for bone. After the formation of the calcified matrix; the cell dies (apoptosis) and is replaced by osteoblasts which brings about the formation of bone

tissue. This completes the endochondral process. (See Figure 3-3 for illustration of ossification process)

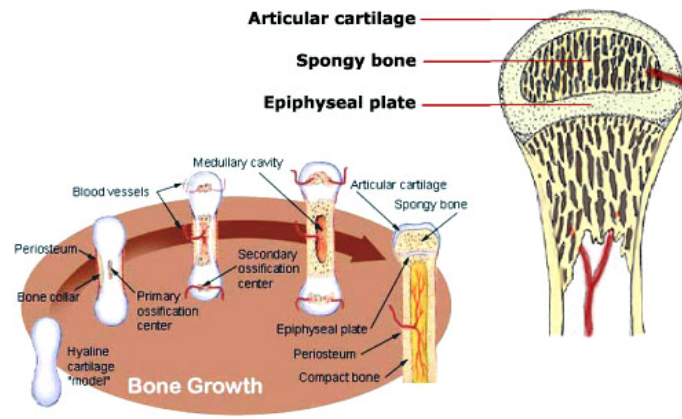


Figure 3-3: Ossification Process – Bone Growth

(l-r) a) five stages of bone growth; b) replacement of cartilage tissue with bone tissue

Ultimately the final formation of bone tissue (the crystal aggregate structure) relies on the cellular aggregation of cartilage tissue. In this liquid state of matter; cells are in flux and are in a constant skirmish for equilibrium (analogous to the study of soap bubbles and laws in understanding surface tension). It is only during the calcification of the cartilage tissue does the aggregation achieve some sort of stable state and informs the final crystalline structural form of the bone tissue.

“... cell and tissue, shell and bone, leaf and flowers, are so many portions of matter, and it is in obedience to the laws of physics that their particles have been moved, moulded and conformed ... Their problems of form are in the first instance mathematical problems, their problems of growth are essentially physical problems.”⁹

There are vast quantities of studies on bones which go beyond the topic of this paper. It would be interesting to note that by the 19th century the formation or structure of bone was directly linked to its function. This theory of “form follows function” of bone

⁹ D’Arcy Thompson, On Growth & Form. (Cambridge: Cambridge University Press. 1961) 7-8

architecture is based on a solid mathematical understanding of principal stress directions in other homogeneous materials. “The idea that the bone was aligned in the direction of stress trajectories was important to the formative stages of thinking about osteon (cellular unit of bone tissue) arrangement and function.”¹⁰

3.2 Inanimate - Foam / Soap Bubbles

“Morphology is not only a study of material things and of the forms of material things, but has its dynamical aspect, under which we deal with the interpretation of the operations of energy.”¹¹

“Matter as such produces nothing, changes nothing, and however convenient it may afterwards be to abbreviate our nomenclature and our descriptions, we must most carefully realize in the outset that the spermatozoon, the nucleus, the chromosomes of the germ-plasma can never act as matter alone, but only as seats of energy and as centers of force.”¹²

It is crucial for this study to understand the concept of surface tension if we want to fully grasp the biological operations of cell packing or aggregation in bone formation; which we discussed in the previous sub-section (surface tension is not exclusive to bone formation but is evident in most growth processes of biological forms). As D’Arcy Thompson evidently states “matter produces nothing”; instead it relies on operations of energy to act upon substances to create morphologies or variations.

The study of surface tension ultimately points us to understand foam or soap bubbles and the forces that act upon them in their formation. Figure 4.2-1 illustrates the general process of foam formation and shows the gradual crystallization structure of foam through the process of self-organization (soap bubbles aim for a state of equilibrium).

¹⁰ R. Bruce Martin, and David B. Burr, Structure, Function, and Adaptation of Compact Bone. (New York: Raven Press. 1989)14.

¹¹ Thompson, On Growth & Form 14.

¹² Thompson, On Growth & Form 14.

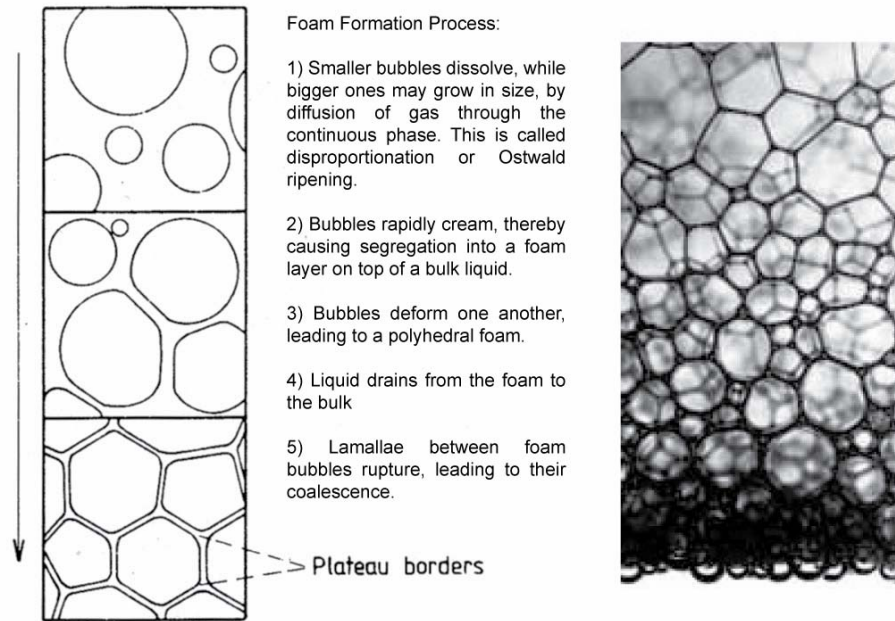


Figure 3-4: Foam Formation Process

(l-r) a) diagrammatic expression of events during formation of a foam layer; b) liquid foam gradient due to gravitational forces

It is of interest to note that foam cells adopt polyhedral shapes which result from the combination of the physical factors governing the foaming process and the geometrical constraints of packing. As observed in Figure 4.2-1, the more pressure is exerted on the cells the more spherical they appear (as seen in the bottom layer of bubbles); polyhedral shapes appear in areas with fewer forces.

3.2.1 Crystallized Structures / Membranes of Foam (Solid)

The crystallized membranes in foams are a result of trapped gas bubbles in a liquid (i.e. beer foam) or solid (i.e. polyurethane foams) state. There are two main classifications of foam: open cell structured foam and closed cell structured foam (Figure 3-5). These two types are distinguished by their porosity; the latter of which has generally a higher compressive strength due to a more solid interior structure.

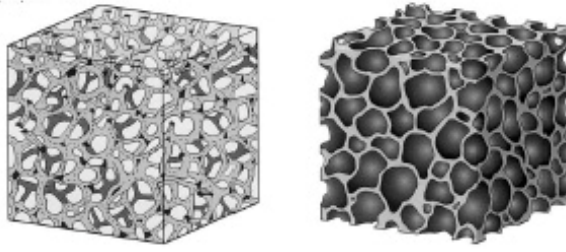


Figure 3-5: Open Cell Structured Foam & Closed Cell Structured Foam

3.2.2 Trapped Air in Soap Bubbles (Void)

Surface tension occurs when “the molecules of the surface layer are being constantly attracted into the interior by such as are just a little more deeply situated; the surface shrinks as molecules keep quitting it for the interior. The process continuous till it can go no further, until the surface itself becomes a minimal area.”¹³ This internal molecular force or pressure inwards is coupled with an outward force or a resistive force to compression which usually is the air pockets within a bubble. Equilibrium between the inward pressure and the outward force is represented by the stabilization of the surface film, having the least possible area (most cases in the shape of a sphere). See Figure 3-6 for illustration.

¹³ Thompson, On Growth & Form 50.

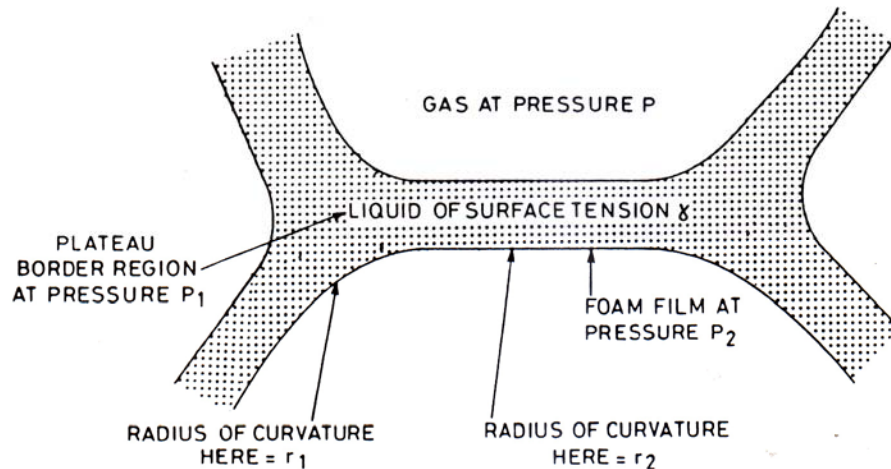


Figure 3-6: Forces Acting Upon a Surface Film Between 4 Soap Bubbles

Aside from these internal molecular forces, the optimization of soap film is dependent on certain controls or conditions (i.e. Frei Otto's minimal path experiments – the direct path indicated by the threads and the needles act as constraints and as boundaries with some room for movement). These conditions include (1) the form of the boundary; (2) the external pressure to which the film is subjected (i.e. gravitational forces in liquid foam)¹⁴.

In summary soap films seek to minimize their surface area, that is, to minimize their surface energy. The optimum shape for an isolated bubble is a sphere given that there are no external constraints acting upon the cells. A Belgian physicist, Joseph Plateau, qualifies these relationships which establish a state of equilibrium for soap films in foam.

Plateau's Rules:

1. Soap films are made of entire smooth surfaces.
2. The average curvature of a portion of a soap film is always constant on any point on the same piece of soap film.

¹⁴ Thompson, On Growth & Form 51.

3. Soap films always meet in threes, and they do so at an angle of 120 degrees forming an edge called a plateau's border.
4. These plateau borders meet in fours at an angle of approximately 109.47 degrees (the tetrahedral angle) to form a vertex

Configurations other than those of Plateau's Rules are unstable and the foam will quickly tend to rearrange itself to conform to these rules.

From these basic understanding of the formation of foam we recognize that there is a solid (crystallized structures or membranes) and a void, which are dependent on each other for their final formation. Through certain idealized rules, due to the natural formation process (self-organizing) of soap films, we can speculate on the emergence of patterns formed by the solid structural membranes. Inherently the ideal packing of voids generates a regularized pattern. But as seen in nature, patterns are affected by other variables (external forces like gravity) which generate multitudes of variable packing and stacking patterns. These then provide a rich field of typologies instinctively relating spatial patterning and its innate structure.

3.3 Geometric - Sphere Packing

Sphere packing problems in math represent the problem of optimum cell-aggregation for a given volume. Mathematicians problematize the arrangement of non-overlapping identical spheres in filling a space with the goal of finding the most optimum outcome. In this section we will examine two main models of sphere packing: face-centered cubic & hexagonal. Packing density is computed as being a ratio of the total volume of spheres (V-sphere) over the volume of the cubic space (V-cube).

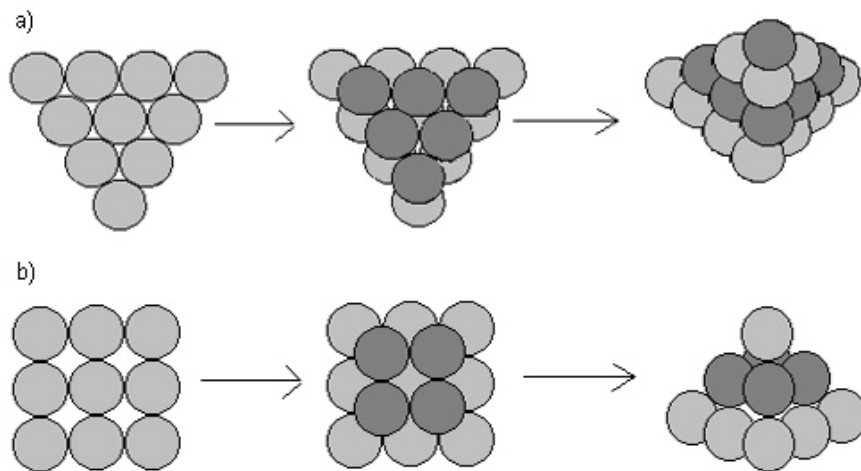


Figure 3-7: Sphere Packing Techniques

(top-bottom) a) face-centered cubic packing - tetrahedral; b) hexagonal packing - octahedral

In face-centered packing the shape of 4 connected spheres take on the form of a tetrahedral; technically the next line of spheres are stacked diagonally from the bottom layer. Hexagonal packing links 5 connected spheres, the form of an octahedral; in this configuration the next layer of spheres are placed in the crevices of the previous layer. These two models of sphere packing are speculated to be the most efficient with a packing density of 0.7405 or 74.05% filled volume.

4 Machining Emergence

In the last two chapters we have looked at various scientific research into natural packing and stacking systems in biological forms; we ended with the mathematical problem of sphere packing. We clearly see that the principle behind the formation of foam (traversing the correlation between solids and voids) is similar to the architectural issues of space planning and structural language. We study these correlations in hopes that architecture, like biological objects, can design forms and structure which are inherent, 'more natural', to its spatial configurations.

In this next chapter we first try to solidify a model of the crystallized structures or membranes which emerge from cellular aggregation through simulation of analogue models. Once these basic modules have been devised we construct transformations or morphologies of these components through digital simulations.

4.1 Packing vs. Stacking

In the following tests, the term packing refers to the two previous techniques discussed in the section on sphere packing (face-centered packing and hexagonal packing, Figure 3-7). The layering of patterns in a packed model occurs in the crevices between spheres in the initial layer. On the other hand, stacking involves the layering of patterns in a more straight or overhead manner; one layer directly over another layer.

4.2 Methodology

➤ Analogue Methodology

The basic set-up of the analogue experiment involves a box 12"x12"x 10", variable sizes of water balloons and regular balloons, and plaster-of-paris. The first step is to inflate the balloons to the desired dimension and arranged in a particular pattern. Then a plaster of paris mixture is prepared; a ratio of 2:1, of plaster to water, is used for the mixture. This is poured into the box of balloons, carefully making sure that the balloons are not floating out of position (the use of water balloons in particular situations prevented floatation). The mold is left to cure for 24 hours; after which it is demolded from the box and the balloons are deflated leaving us with the crystalline imprint of the negative spaces.

The goal of the analogue experiments is to physically understand the interplay between solid (the crystallized membranes bridging the empty spaces) and voids (the inner air pockets which exist in the balloons) in 3-dimensional space. This will hopefully generate an initial typology of members.

➤ Digital Methodology

A digital simulation is used to explain the process in representing a member which materializes from the analogue model. The initial step is to create a solid model of the physical experiment. A lattice grid is used to identify a member and its relation to its neighboring members. The lattice also locates the nodes of the members which are then used in the creation of a digital model of a member.

4.3 Analogue Experiments

4.3.1 Homogeneous Packing

4.3.1.1 “Face-Centered” Packing, Triangular Grid

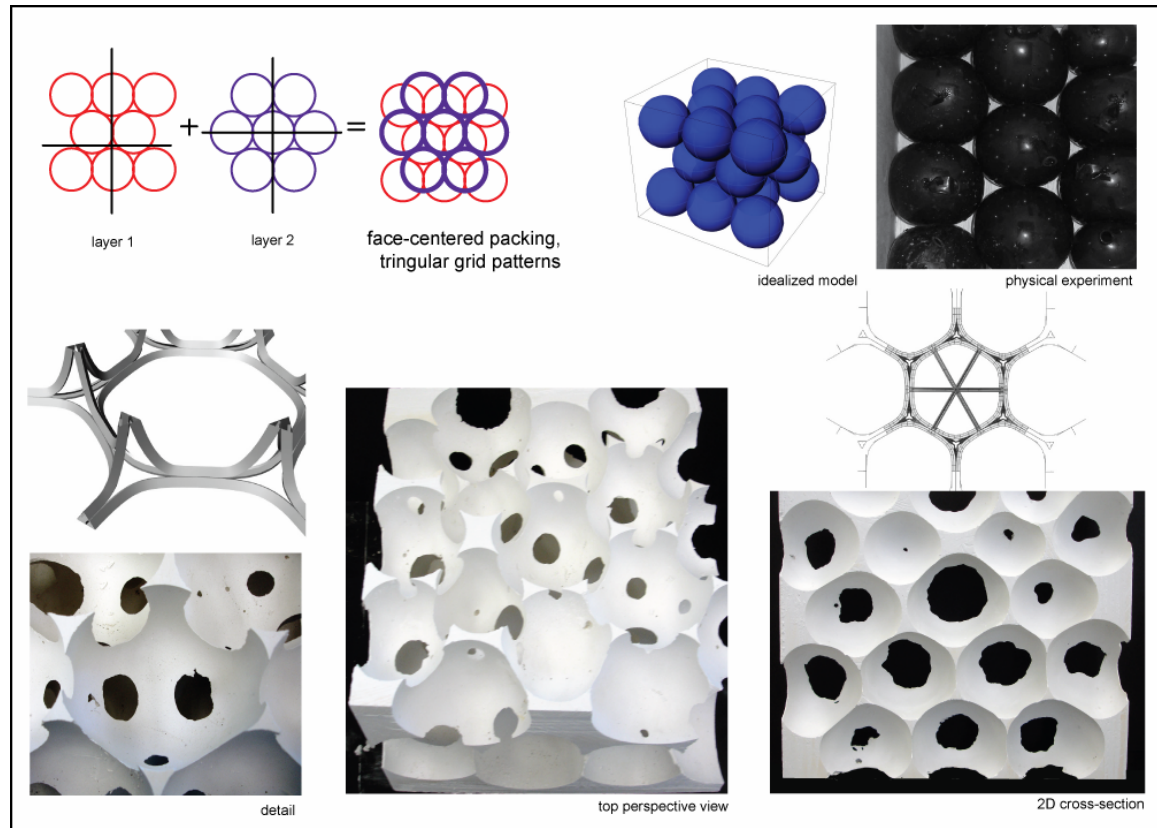


Figure 4-1: Experiment 1.1 _ Face-Centered Cubic Packing

Experiment 1.1 is a homogeneous packing of spheres in the “face-centered cubic packing” pattern (tetrahedral pattern). Three layers of balloons were stacked fitting 46 balloons in a 12”x12”x10” grid. More balloons were fitted in one layer due to the diagonal packing or placement of spheres. Each member staggers to meet the bottom portion of a member on the next layer (keeping in mind that balloons on the second layer are stacked between the crevices of 3 balloons in the first layer).

The results of this experiment are a consistent 3-sided member which bridges between spherical voids. It is also of interest that the cross section through a node produces a triangular section.

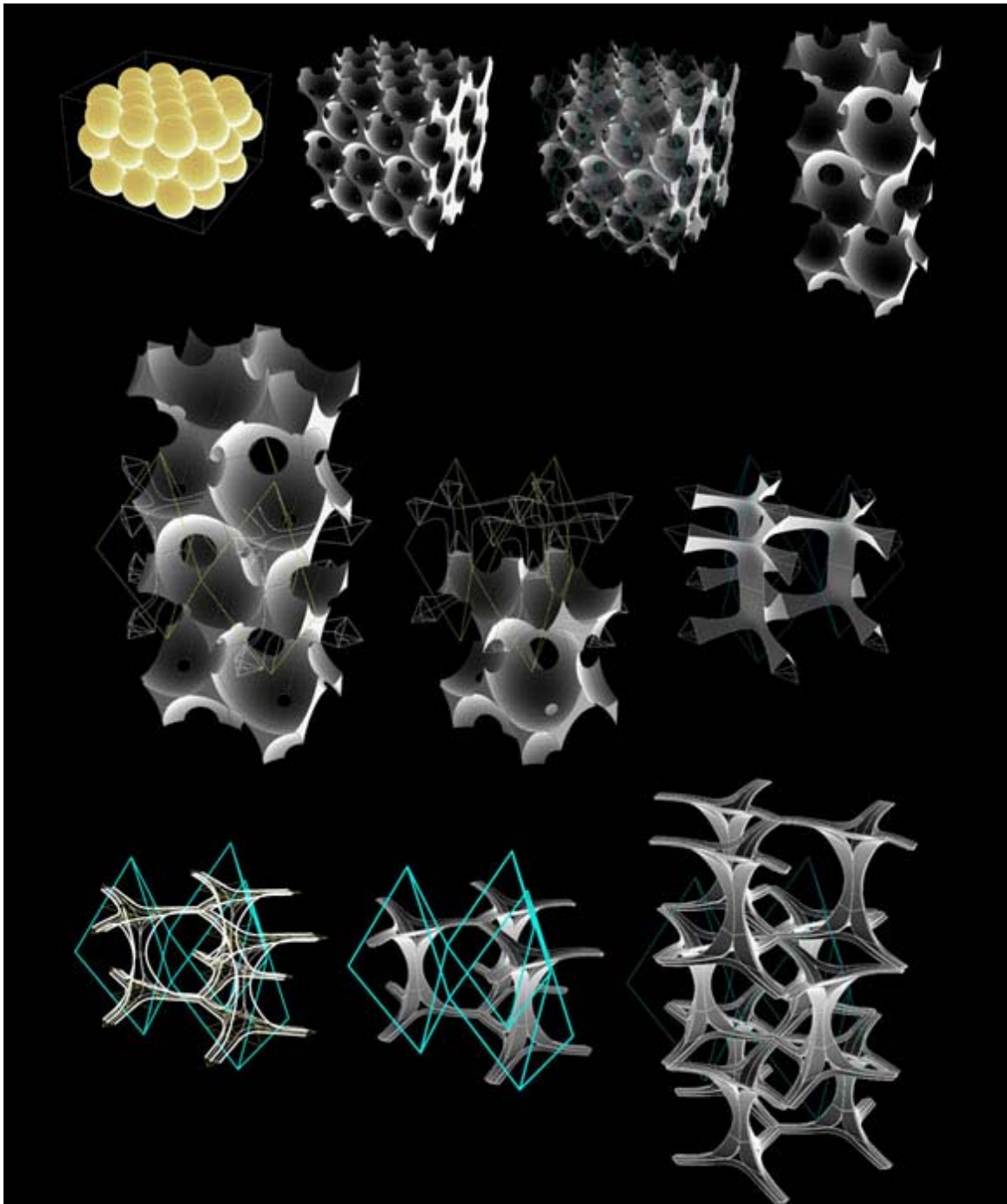


Figure 4-2: Experiment 1.1 _ Digital Process Illustrating the Reconstruction of Members

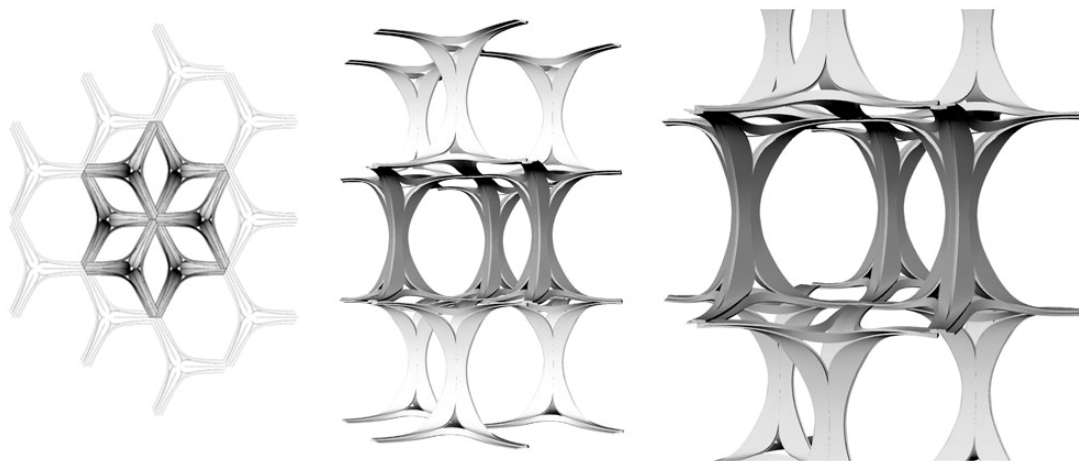
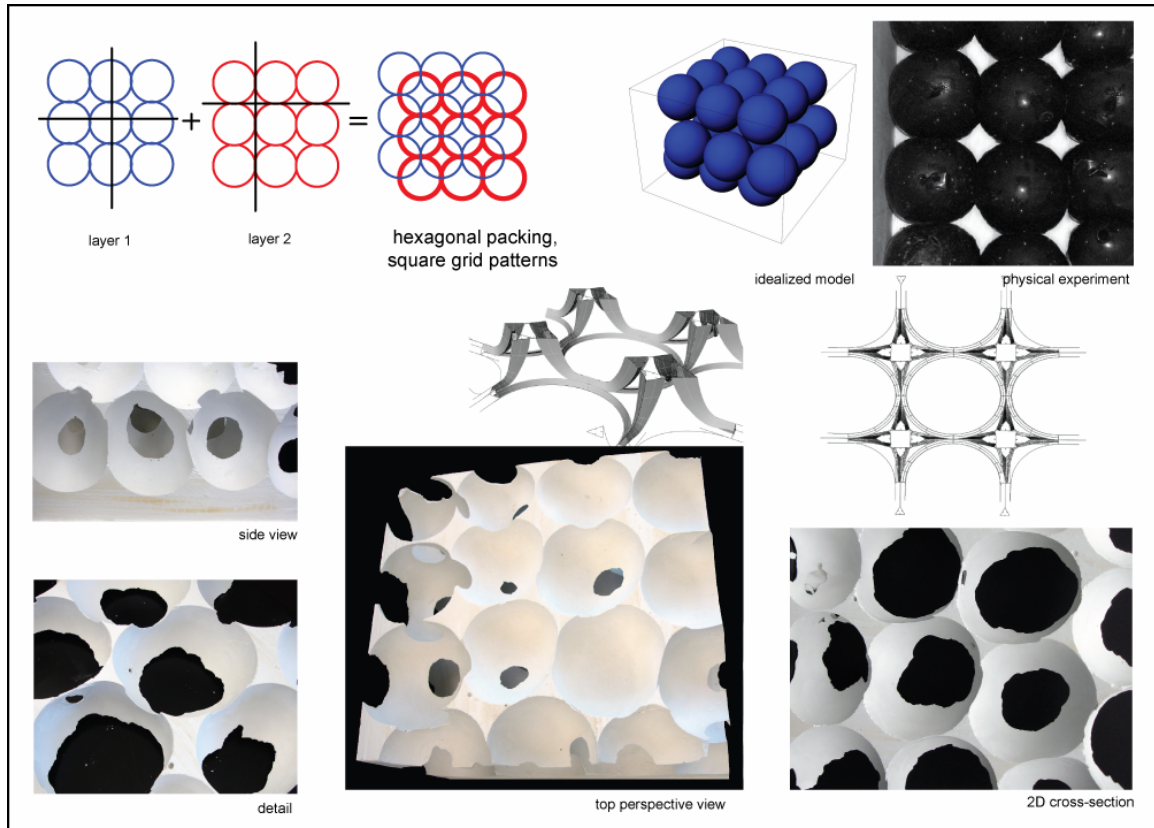


Figure 4-3: Experiment 1.1 _ 3-Sided Members As Connection Between Voids

4.3.1.2 “Hexagonal” Packing, Square Grid



Experiment 1.2 is also a homogeneous packing of spheres in a “hexagonal packing” pattern (octahedral pattern). Three layers of balloons were packed, fitting 44 balloons in a 12”x12”x10” grid.

The results consist of a 4-sided member aggregating to form the crystalline structure. The cross-section per member shows a 4-sided section.

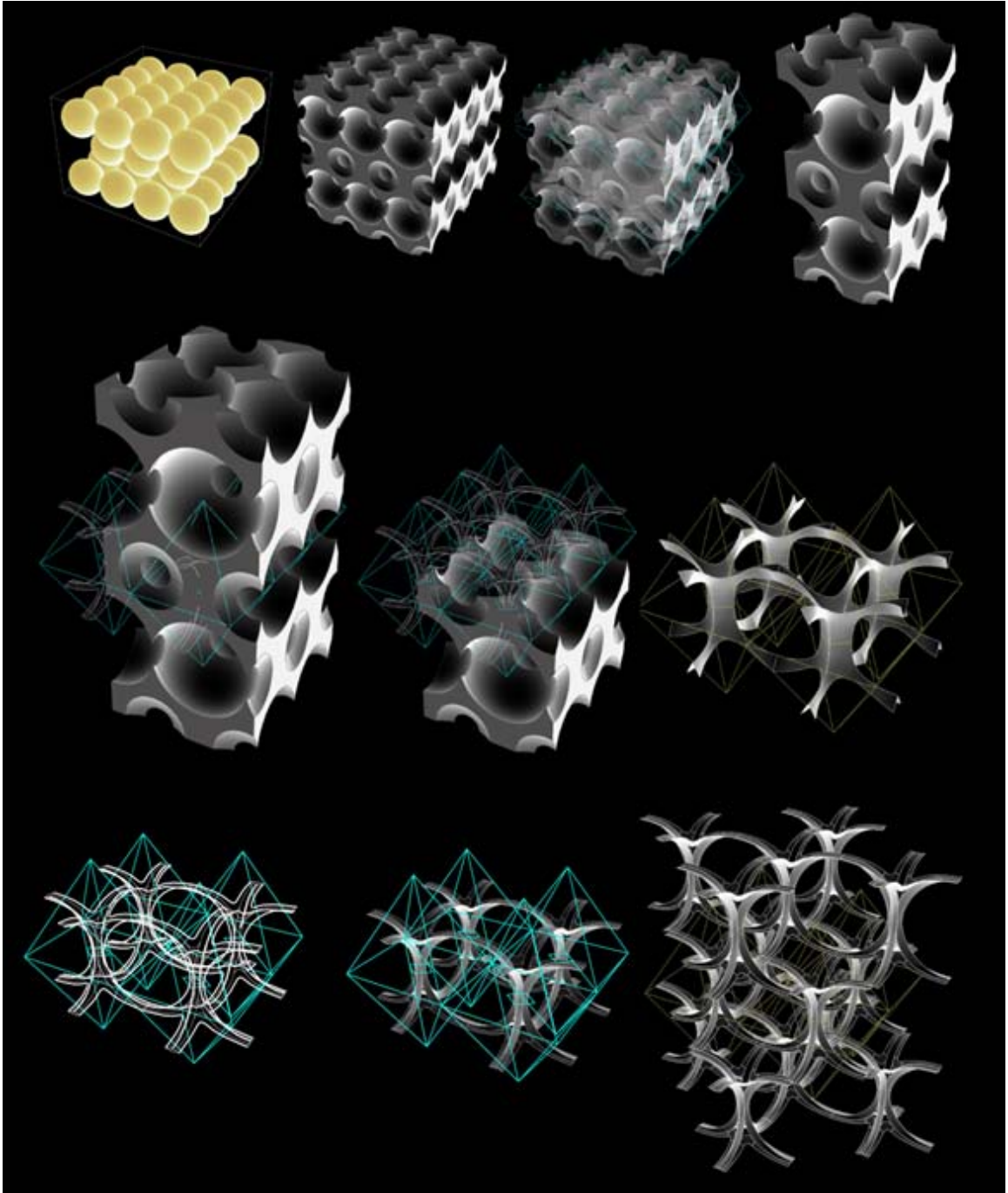


Figure 4-5: Experiment 1.2 _ Digital Process Illustrating the Reconstruction of Members

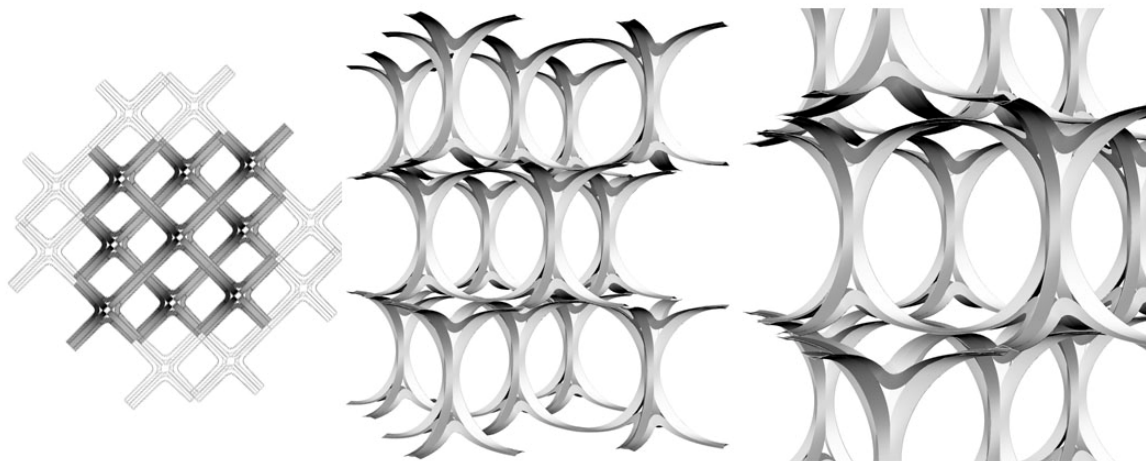


Figure 4-6: Experiment 1.2 _ 4-Sided Member As Connection Between Voids

4.3.1.3 “Hexagonal” Packing, Square Grid, Dense

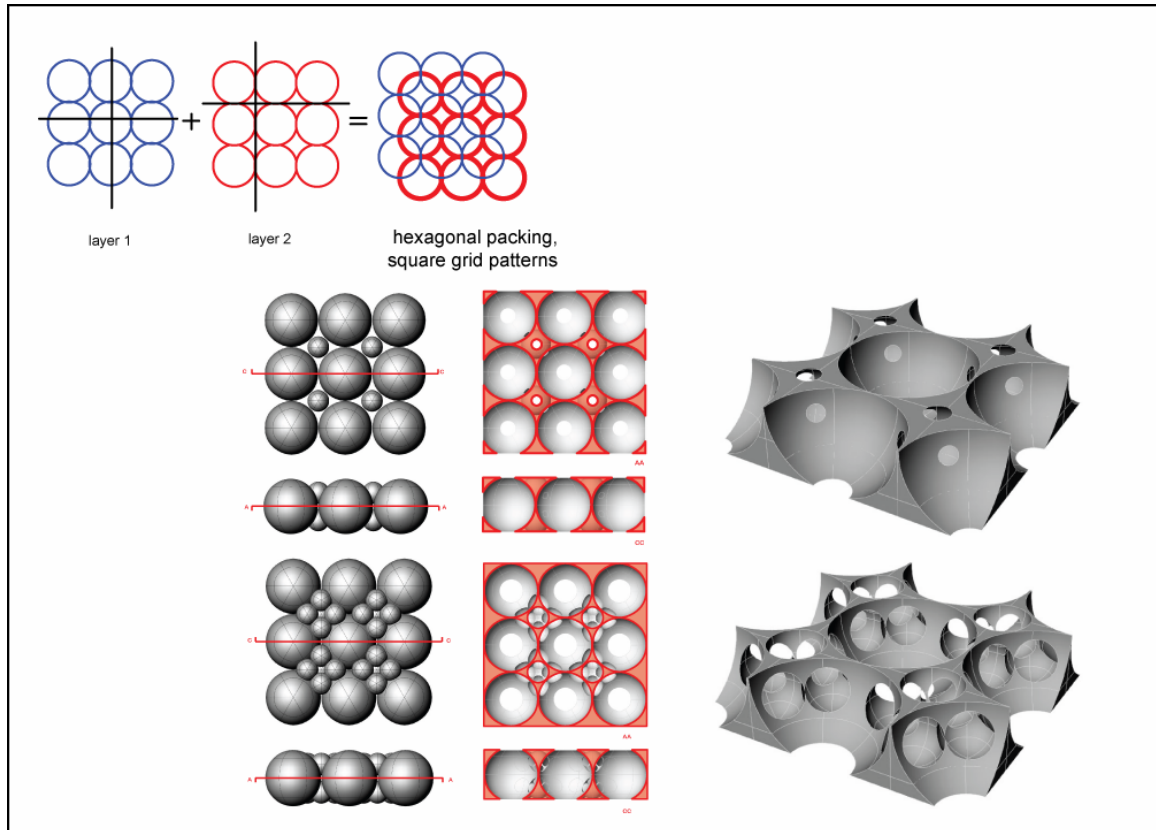


Figure 4-7: Experiment 1.3 _ Hexagonal Packing - Dense

Experiment 1.3 is a digital experiment in understanding how the members change due to the addition of smaller spheres in void spaces created by the packing of larger spheres in a “hexagonal packing” pattern.

The initial process was to digitally synthesize the homogeneous packing in experiment 1b. Next we added smaller spheres in voids between the larger spheres. The smaller spheres were approximately 50% (volume) less than the larger spheres.

Two simulations were conducted; the first added 8 smaller spheres per layer and next adding 32 smaller spheres per layer.

A cross section perspective view was cut to understand the augmentation to the original 4-sided member. These images show an ever increasing lighter membrane by the reduction of inner volumes.

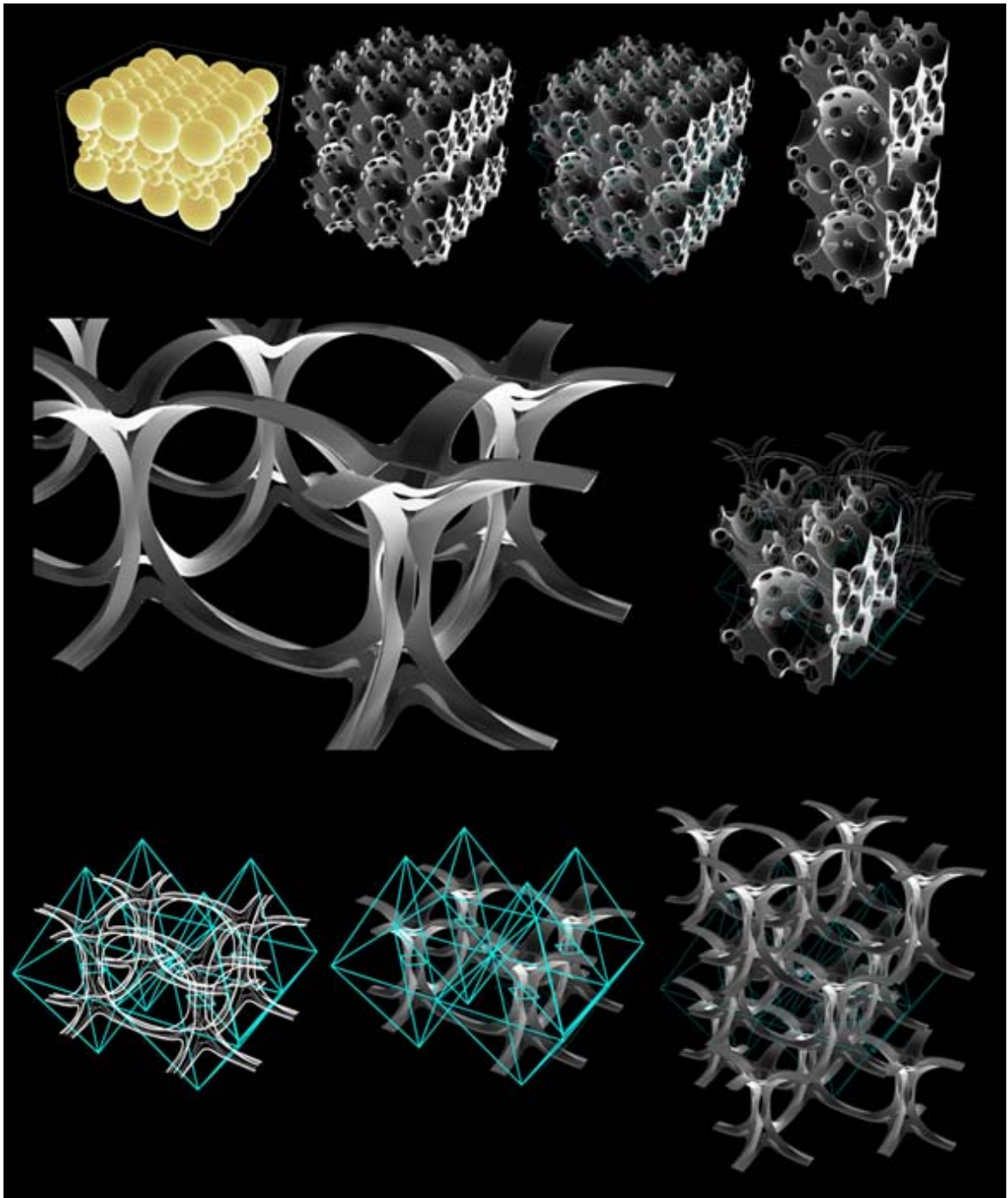


Figure 4-8: Experiment 1.3 _ Digital Process Illustrating the Reconstruction of Members

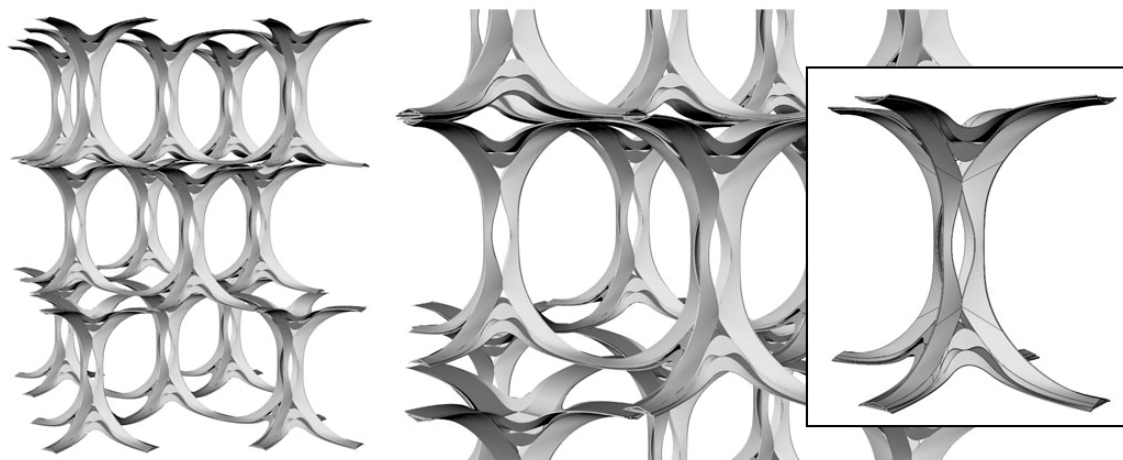


Figure 4-9: Experiment 1.3 _ 4-Sided Dense Member

4.3.1.4 Summary of Members

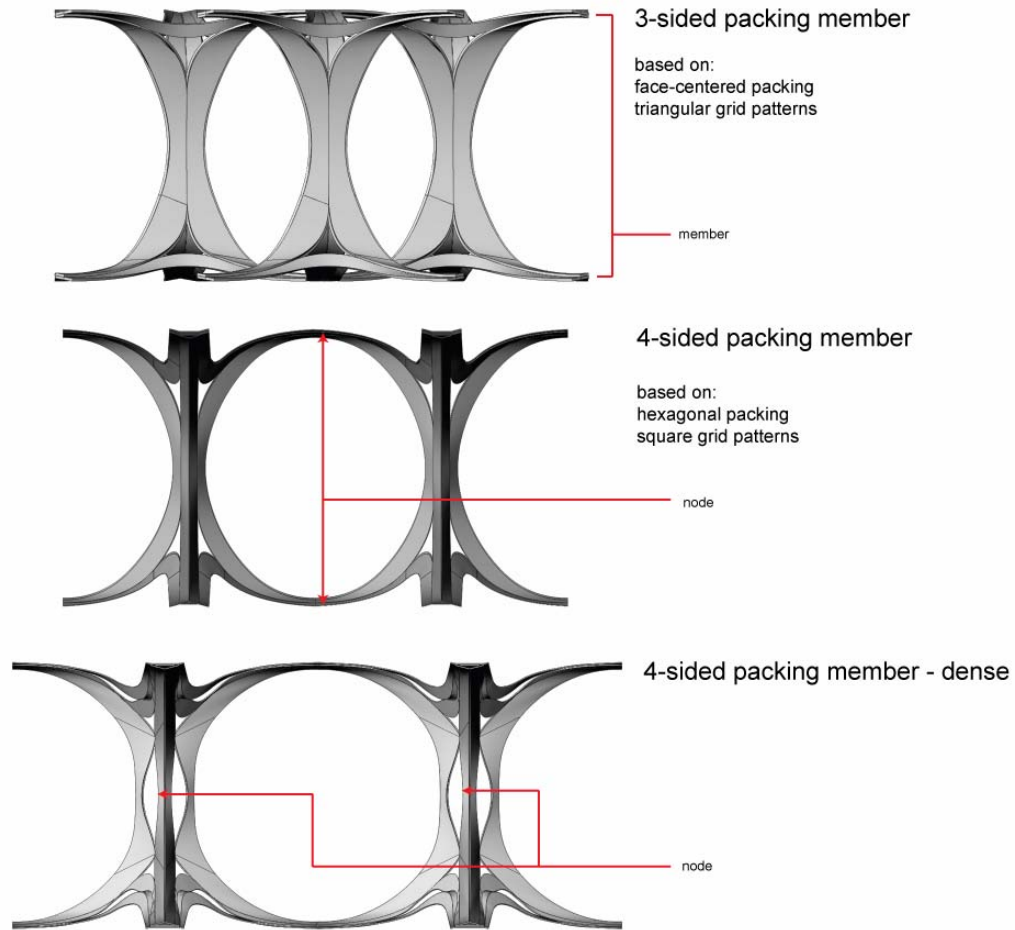


Figure 4-10: Summary of Members from Packing Strategies

The resultant members are wider in the corners which can be attributed to the direct load acting on the member by the packed sphere directly above it. There is also a horizontal zigzag pattern which appears when we look at the members in elevation. This is due to the pattern of packing which optimizes spatial configurations by placing spheres on the crevices of the bottom layer.

4.3.2 Homogeneous Stacked

4.3.2.1 Triangular Grid, Straight Stacked

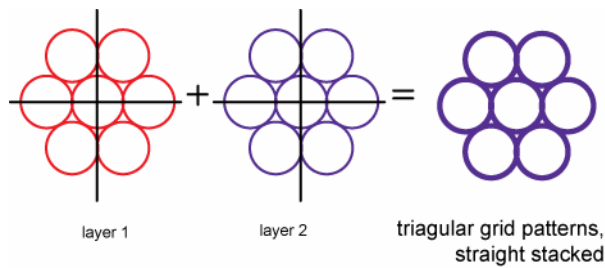


Figure 4-11: Experiment 2.1 _ Triangular Grid Pattern in a Straight Stack

Experiment 2.1 is a homogeneous stacking of spheres using a triangular grid or pattern as its initial layer. The succeeding layers are stacked direct above the initial layer. This resulted in the creation of 3-sided members. The cross section through the node is a diamond shape; this cross-section emerged due to the merging of the triangular cross-section of a bottom member and a top member.

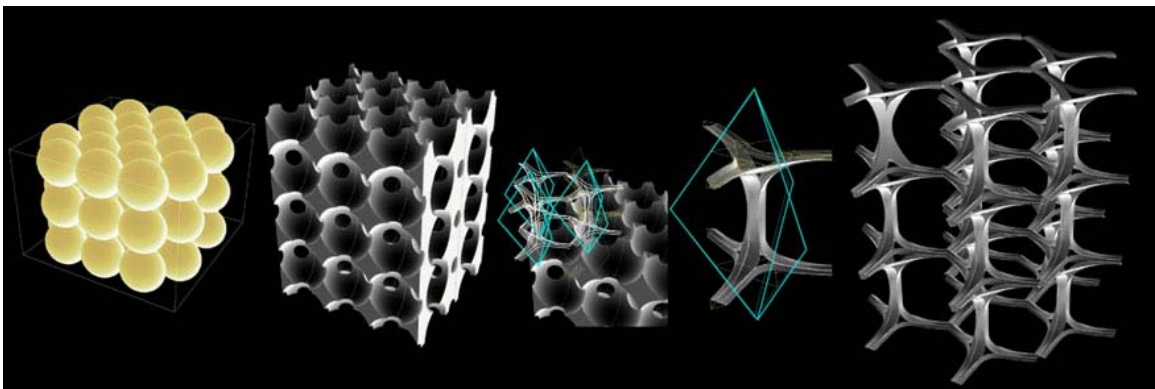


Figure 4-12: Experiment 2.1 _ Digital Process Illustrating the Reconstruction of Members

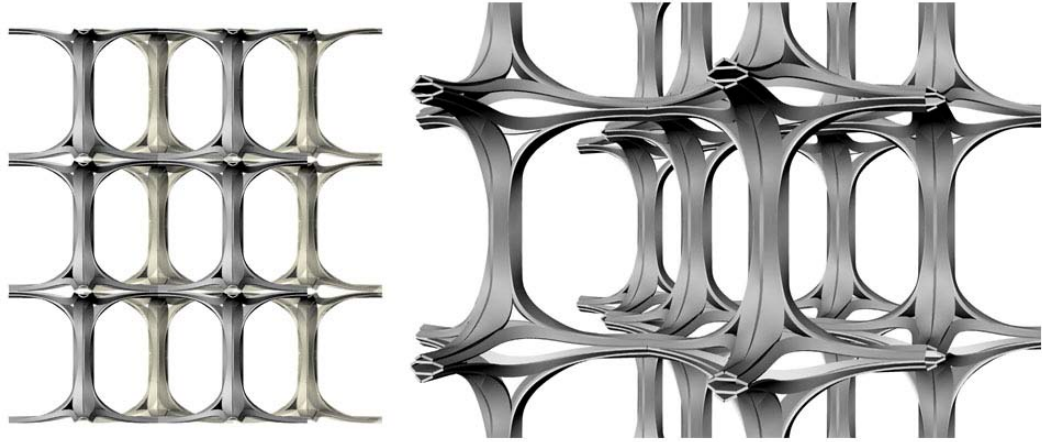


Figure 4-13: Experiment 2.1 _ 3-Sided Members As Connection Between Voids

4.3.2.2 Square Grid, Straight Stacked

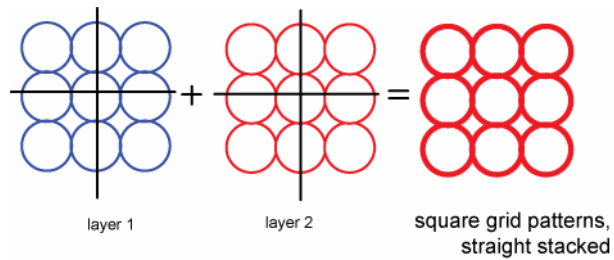


Figure 4-14: Experiment 2.2 _ Square Grid Patterns in a Straight Stack

Experiment 2.2 is a homogeneous stack consisting of the layering of square grids. The result of this experiment is similar to experiment 2.1 with the exception that they form a 4-sided member to traverse the connections.

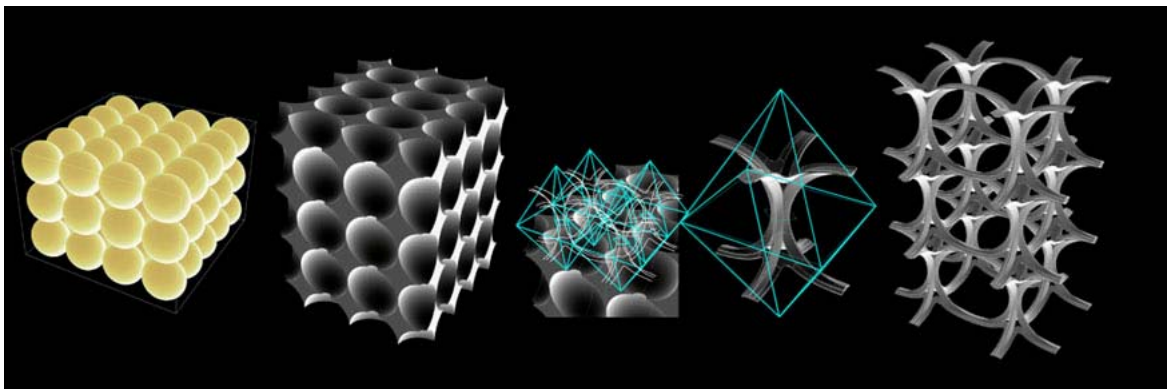


Figure 4-15: Experiment 2.2 _ Digital Process Illustrating the Reconstruction of Members

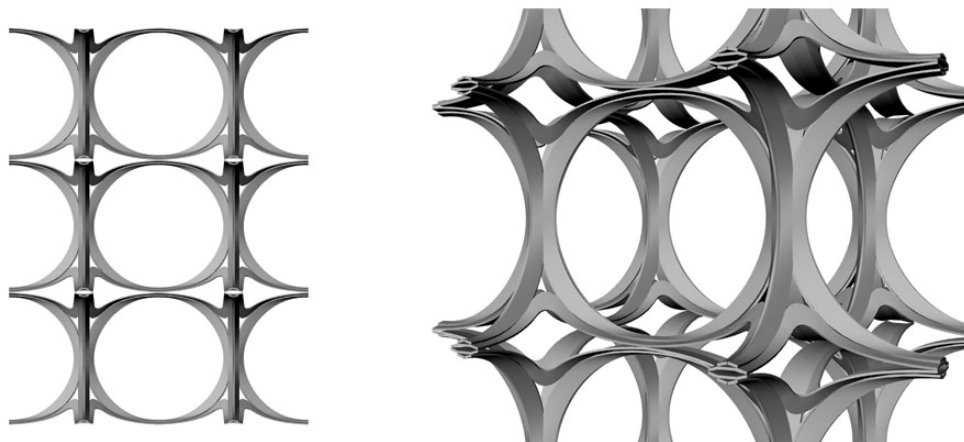


Figure 4-16: Experiment 2.2 _ 4-Sided Members As Connection Between Voids

4.3.2.3 Summary of Members

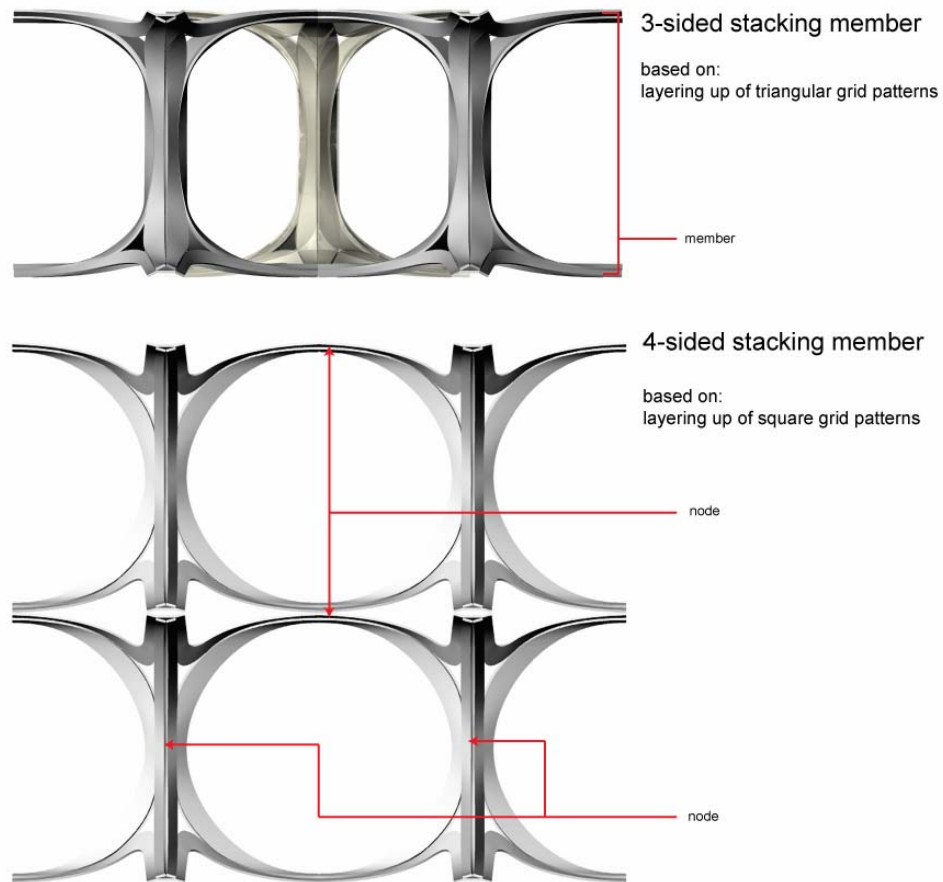


Figure 4-17: Summary of Members from Stacking Strategies

The ensuing members have diamond shaped cross-sections at its nodes. This is again due to the direct stack of one member (which has a triangular cross-section on its own) with another member. These, consequently, thickens the connective arches between members which is consistent with our understanding of its physical properties. Since, spatially, one sphere is directly on top of each other the arch receives more load.

4.3.3 Heterogeneous / Mixed

4.3.3.1 Radom Sized Spheres, Variable Method of Tangency (i.e. packed, stacked)

Experiment 3.1 is a heterogeneous mix of spheres using variable sphere sizes within a grid of 12"x12"x10". The results of these experiments are a mixture of 3-sided and 4-sided members which is shown in detail in experiment 1 & 2. The following are digital models of a typical combination between a 3-sided and a 4-sided member.

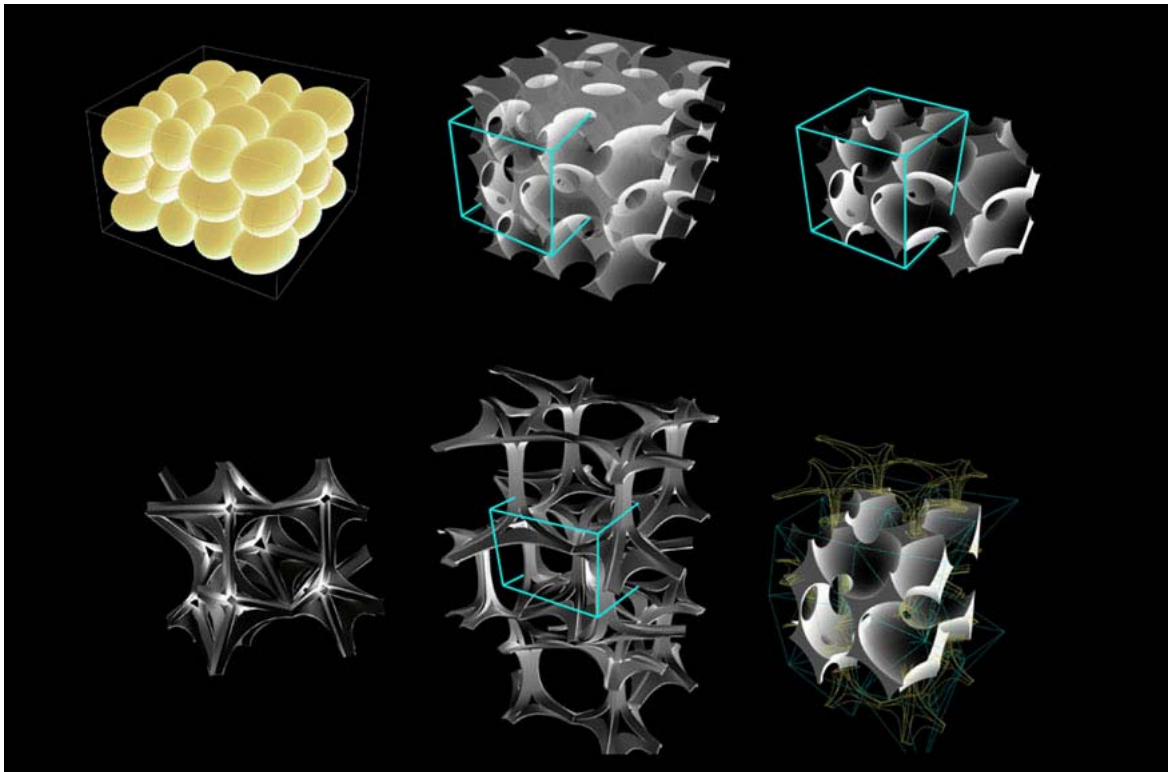


Figure 4-18: Experiment 3.1 _ Digital Process Illustrating the Reconstruction of Members

It should also be noted that one aspect of flexibility within this members is its ability to split any one of its branches to meet a near-by connection.

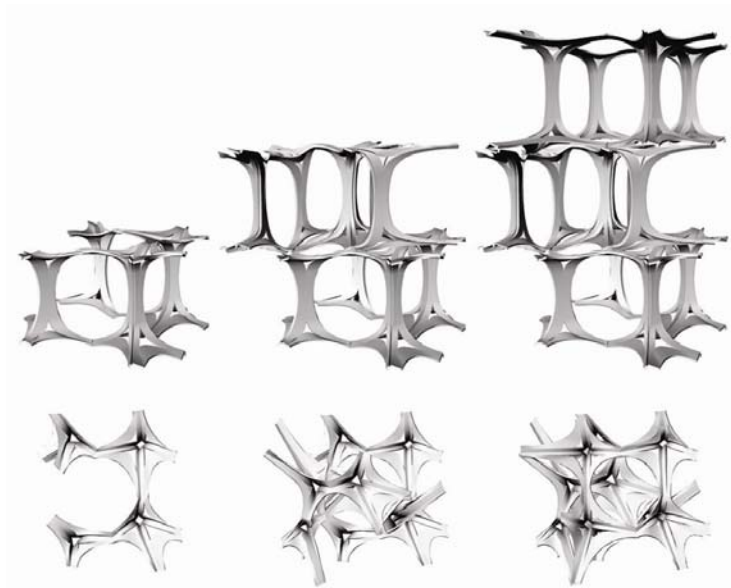


Figure 4-19: Experiment 3.1 _ Random Packing & Stacking Creates Variable Member Patterns

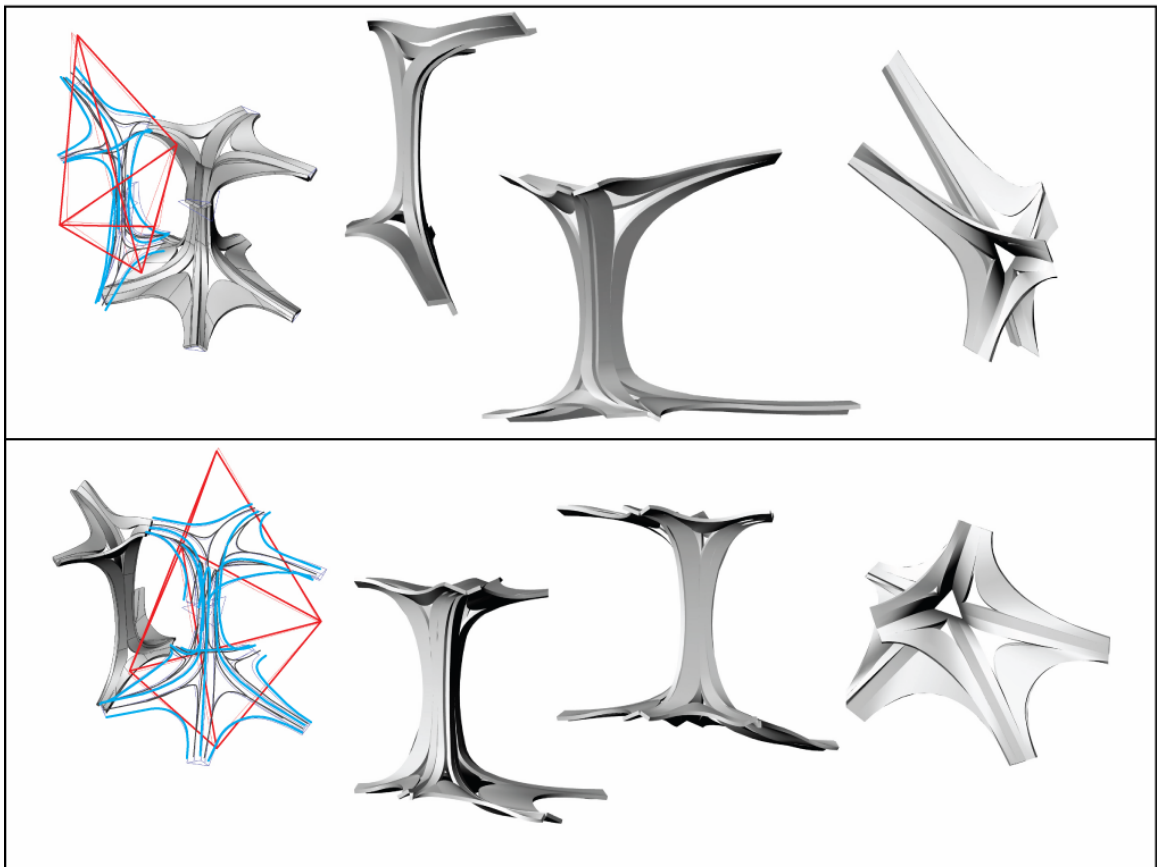


Figure 4-20: Experiment 3.1 _ 3-Sided Member Meets a 4-Sided Member

4.3.3.2 Packed Set of Spheres + Anomalous Object

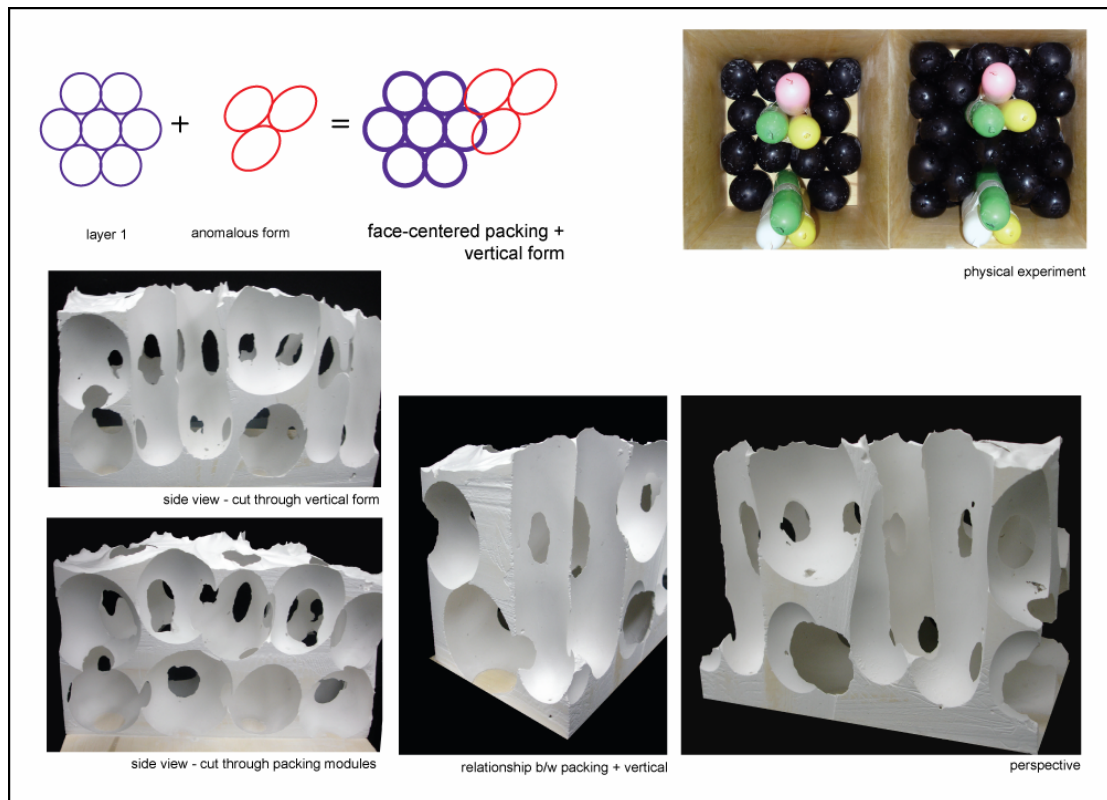


Figure 4-21: Experiment 3.2 _ Packing + Vertical Anomalous Object

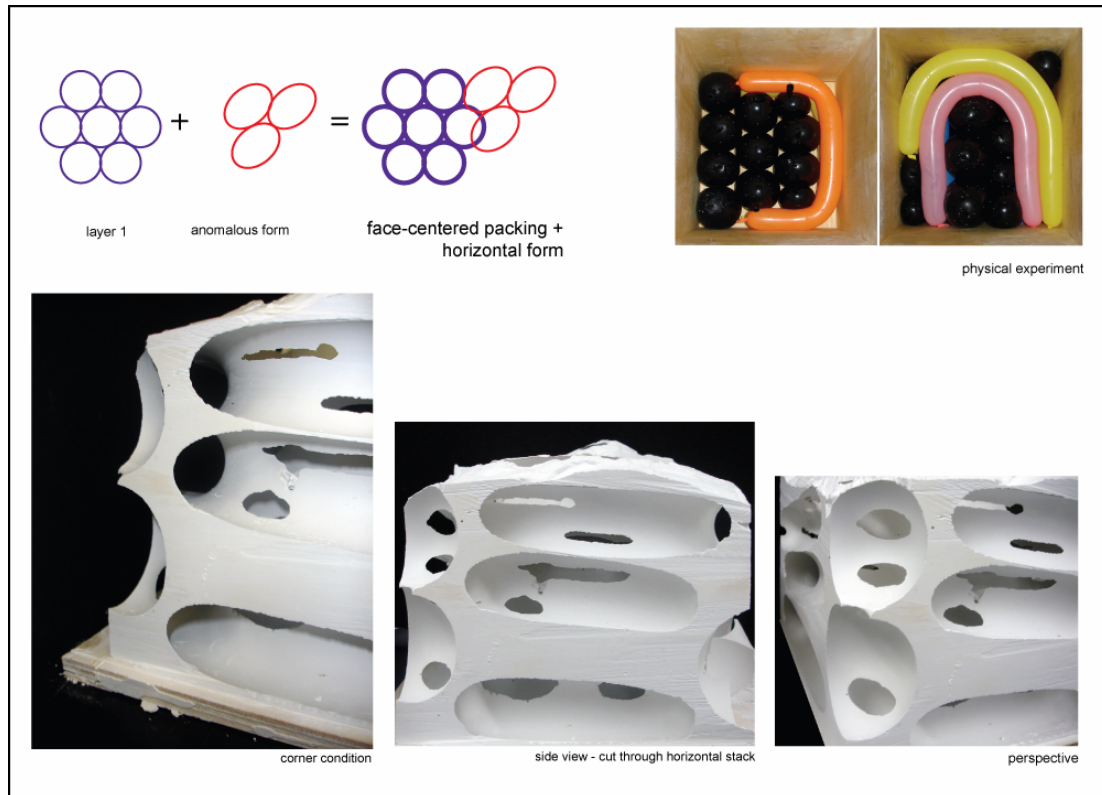


Figure 4-22: Experiment 3.3 _ Packing + Horizontal Anomalous Object

Experiment 3.2 and 3.3 are experiments which show how members react when there is a large vertical member running through a homogeneous packing pattern. The first experiment has two groups of vertical members (simulated by the use of 3 long balloons) placed within the box. This is then surrounded by packed spheres arranged in a “face-centered cubic” pattern. The second experiment is similar to the first with the exception that the “face-centered cubic” spheres need to traverse horizontal members running along the length of the box.

Basically, what we see is that in a 3-sided member, the flat face is oriented towards the vertical stack creating the side wall of the vertical or horizontal member.

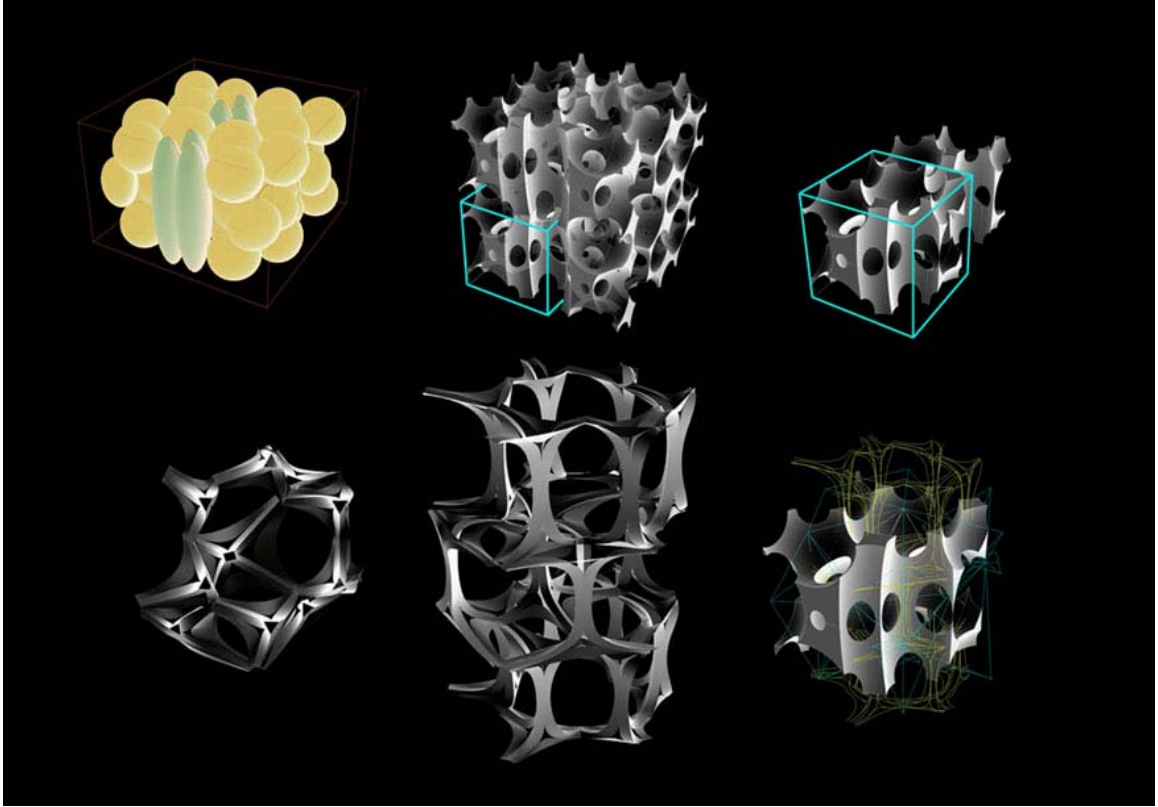


Figure 4-23: Experiment 3.2 _ Digital Process Illustrating the Reconstruction of Members

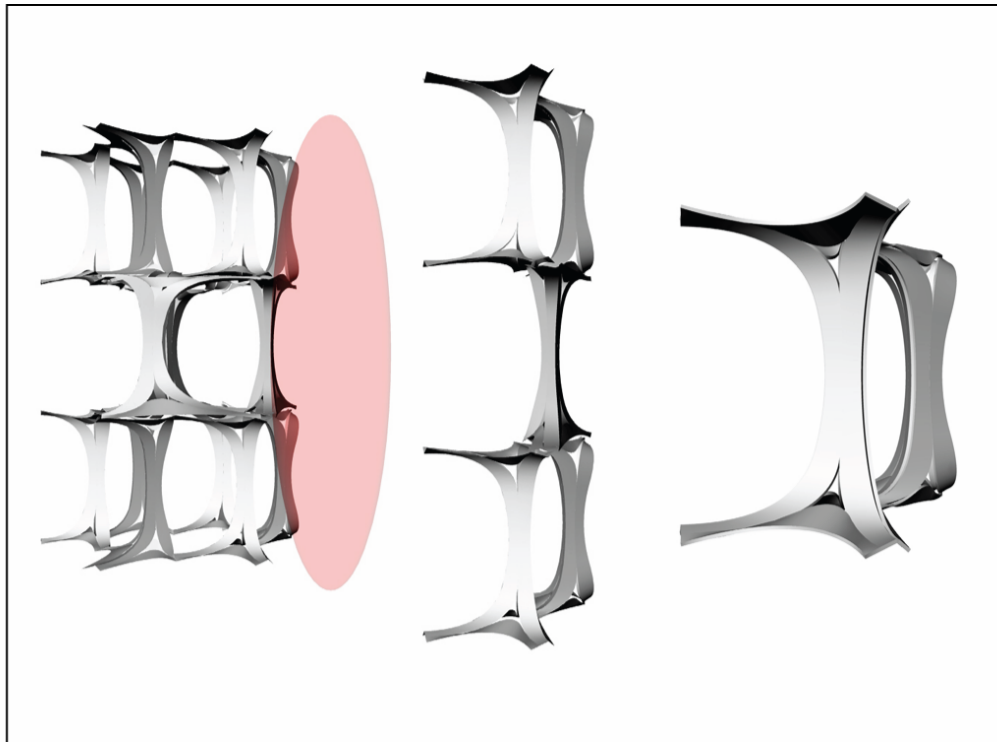


Figure 4-24: Experiment 3.2 & 3.3 _ Flat Face of Member Creates Surface Boundary of Aperture

4.3.4 Summary of Findings

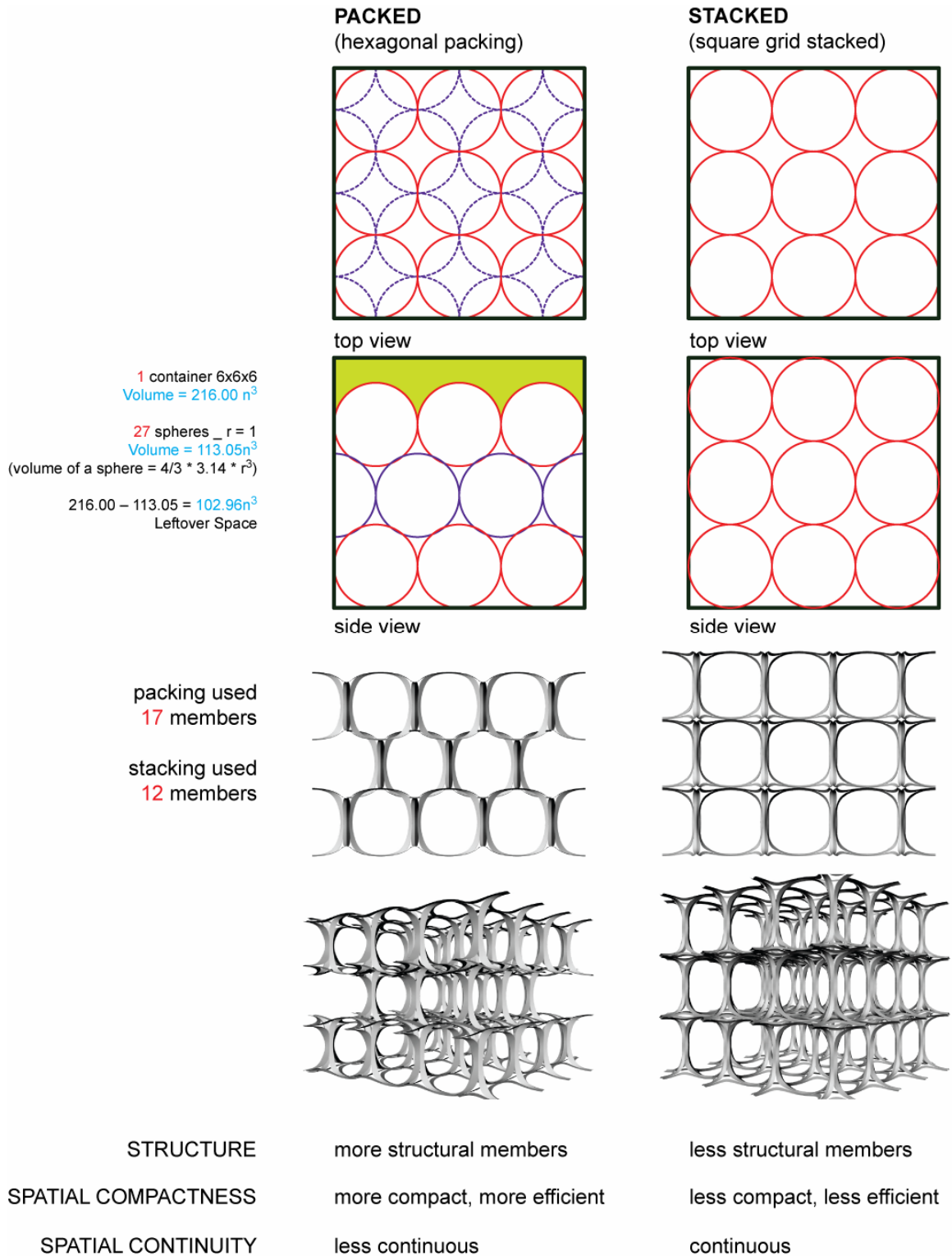


Figure 4-25: Summary of Findings from Analogue Experiments

In figure 4-25, a general comparison is presented between packing strategies and stacking strategies. Packing members are wider in its corners and receive most of its loads directly above each member. This creates a more arched column configuration and exhibits less spatial continuity or horizontality. Aside from a more arched column member, a series of packing members creates a more compact spatial organization (voids) by using more structural members (solids).

In a stacked system, the members are more orthogonal and are generally thicker in the nodal area; this is primarily where the loads are concentrated. In comparison to a packed system, a stacked system is less efficient in terms of spatial organization but uses less structural members and creates more spatial continuity or horizontality.

The formation of heterogeneous or randomly arranged spaces uses a mixture of packing and stacking strategies or a hybrid system. These systems are bridged by varying types of member (i.e. 3-sided, 4-sided members, etc...). The strategy adopted in adjoining a large open space or meeting a corner is by strategically placing or reorienting the members so that the flat-face is facing the opening.

4.4 Grid / Pattern Layering

4.4.1 Vertical Relationships

Vertical transformations of members are dependent on the spatial relationships in layering up of grid patterns, “in between” nodes are related based on proximity. These relations create transformations in the vertical connectivity of members.

There are 6 examples presented. We’ve encountered the first 4 patterns in the previous section. The last 2 are hybrids of a triangular grid and a square grid.

4.4.1.1 Face-Centered Packing

In packing spheres, the layering of grids is tangent in the crevices of the previous layer. The vertical relationship between nodes (indicated by the vertical red arrow) forms a zigzag pattern; wherein one sphere will never be directly above another sphere.

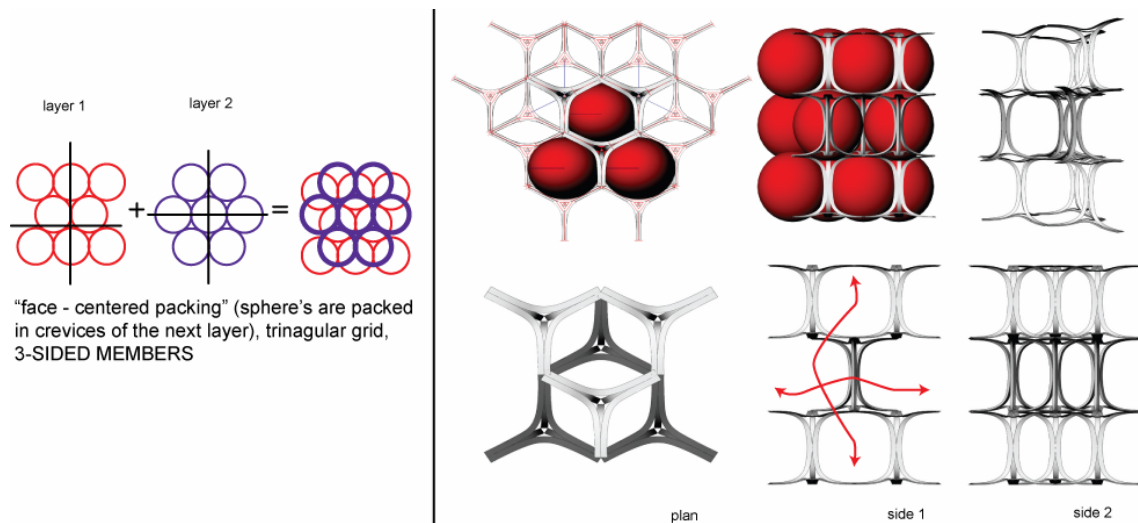


Figure 4-26: Vertical Relationship Face-Centered Packing
(l-r) a) diagram of patterns; b) plan, elevation, and perspective views

4.4.1.2 Hexagonal Packing

The vertical relationship of the hexagonal packing is similar to the flows shown in the face-centered packing. The variation in this instance is the node itself, being a 4-sided member.

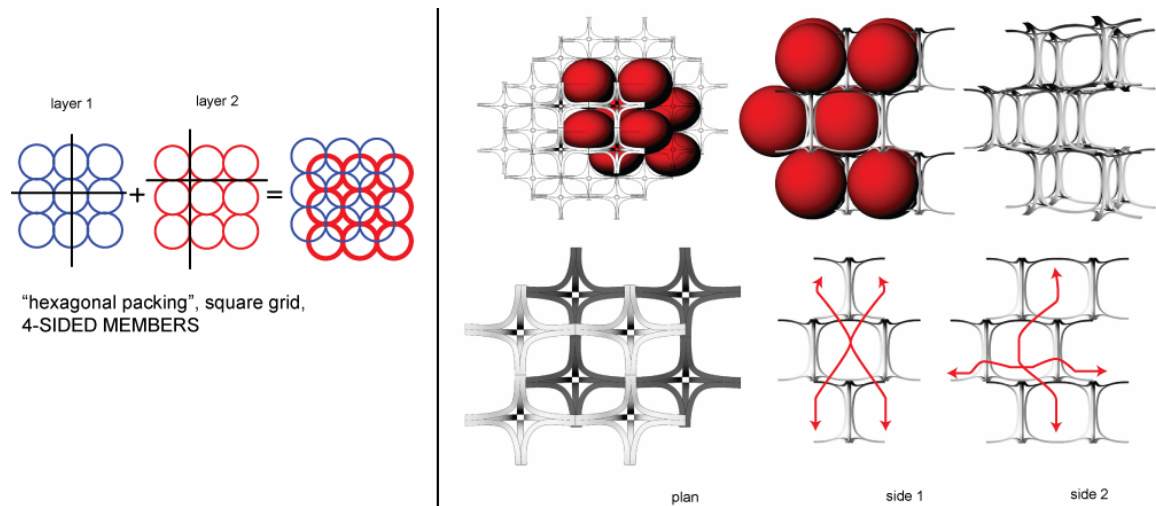


Figure 4-27: Vertical Relationship _ Hexagonal Packing
(l-r) a) diagram of patterns; b) plan, elevation, and perspective views

4.4.1.3 Triangular Grid Stacked

In a stacking problem of triangular grids there is no zigzag relationship between members. Instead a straight vertical or horizontal relationship exists. In plan view, a straight relationship is also observed but occurring diagonally.

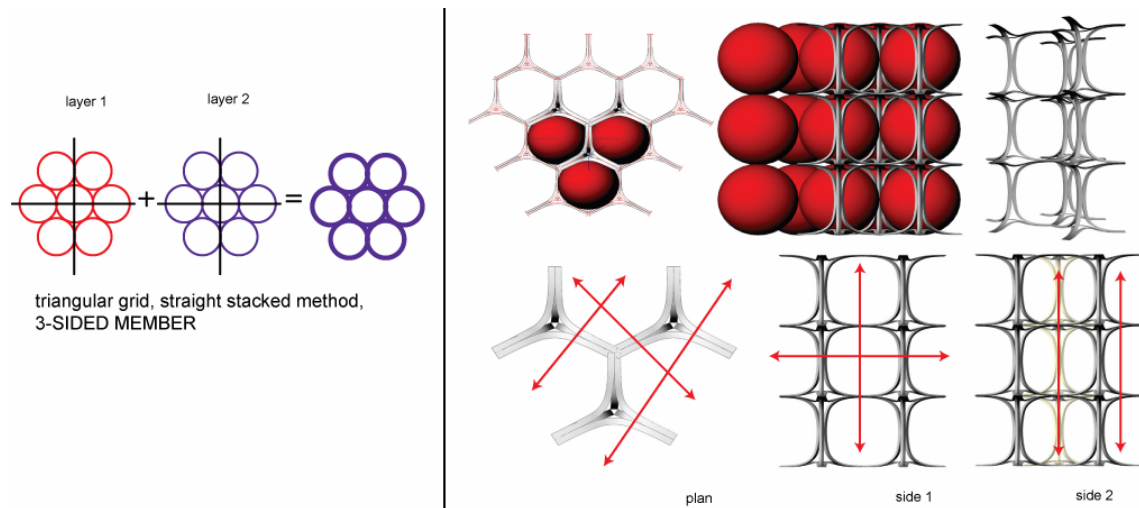


Figure 4-28: Vertical Relationship _ Triangular Grid Pattern Stacked

(l-r) a) diagram of patterns; b) plan, elevation, and perspective views

4.4.1.4 Square Grid Stacked

Stacked spheres are laid up one on top of the other creating typical column connections. Similar to the previous example, the square stacked grid has straight vertical and horizontal relationships. The relationship in plan also occurs in a direct cross fashion.

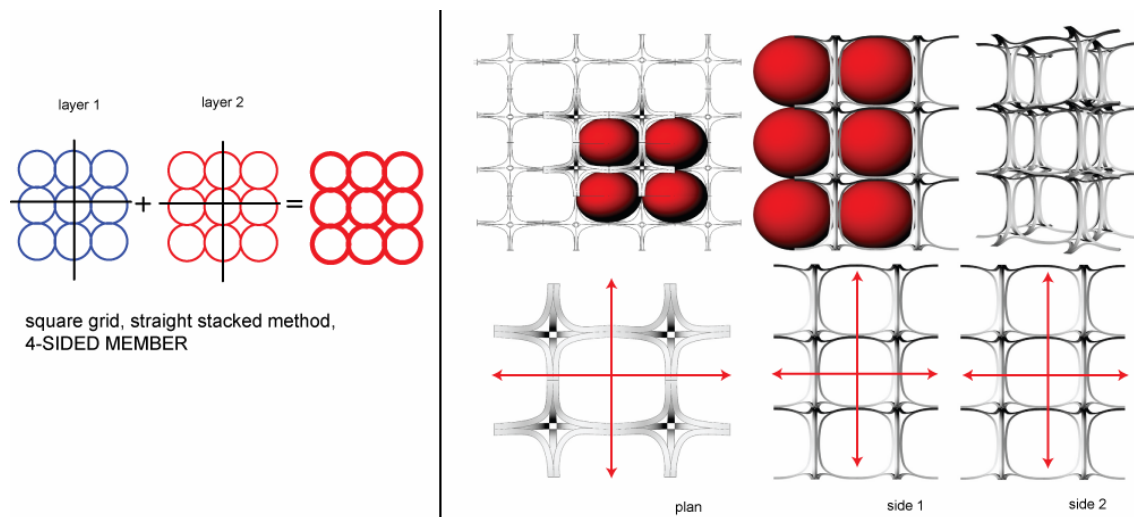


Figure 4-29: Vertical Relationship _ Square Grid Pattern Stacked

(l-r) a) diagram of patterns; b) plan, elevation, and perspective views

4.4.1.5 Triangular + Square, variable method for tangency

The layering up of a square grid over a triangular grid gives us a hybrid pattern which creates interesting relationships in generating the member. There is no fixed method for creating the members. Although the experiment shows us that given the same sized spheres the relationship and resulting form of the members is sometimes partially packed, or stacked.

There is also a gradual straightening of vertical relationships, as indicated by the red arrows on the side elevation view.

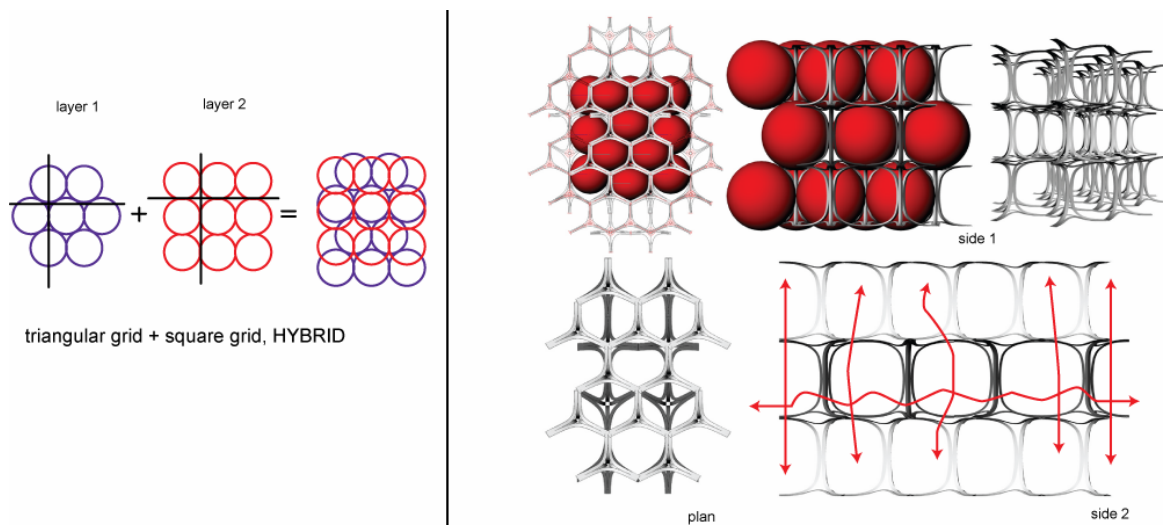


Figure 4-30: Vertical Relationship _ Triangular Grid + Square Grid

(l-r) a) diagram of patterns; b) plan, elevation, and perspective views

4.4.1.6 Square + Triangular, variable method for tangency

This experiment moves from a square grid to a triangular grid. Similar to the previous layer there is a gradual straightening of the vertical relationships between members. There is also a horizontal zigzag relationship between members which is consistent with the patterns we saw previously in the vertical relationships of packing systems.

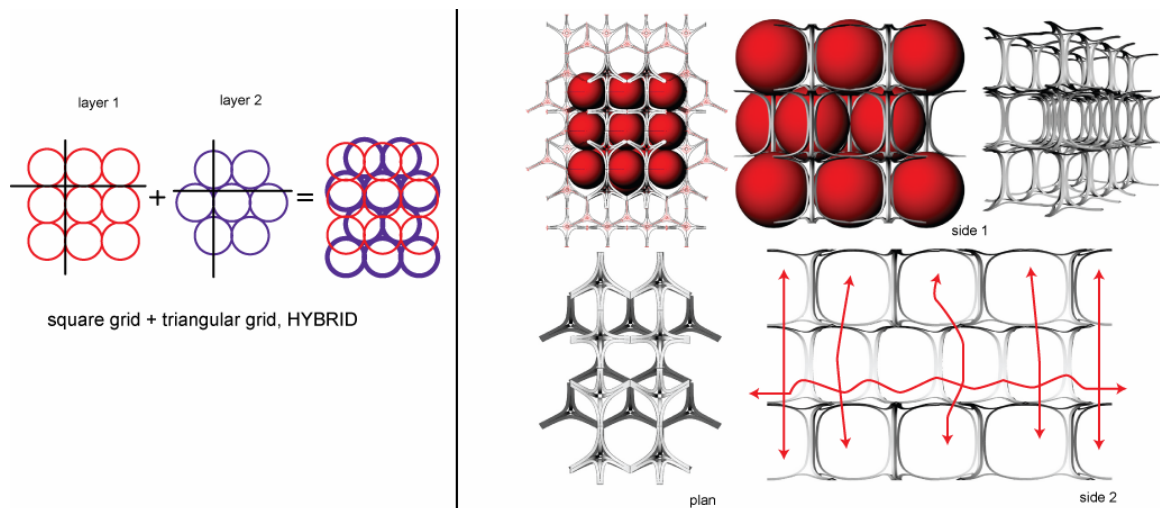


Figure 4-31: Vertical Relationship _ Square Grid + Triangular Grid

(l-r) a) diagram of patterns; b) plan, elevation, and perspective views

4.4.2 Vertical Transformations

Taking the 6 pattern relationships we can imagine them transforming from 1 type to another. Within each transformation there is an infinite number of instances (...n = instance) which form different types of members; given a more gradual transformation. The matrix below shows the different layering possibilities. The arrows per transformation indicate the general movement that needs to happen for the transformation to occur. Figure 4-31 is a matrix which illustrates the varying transformations

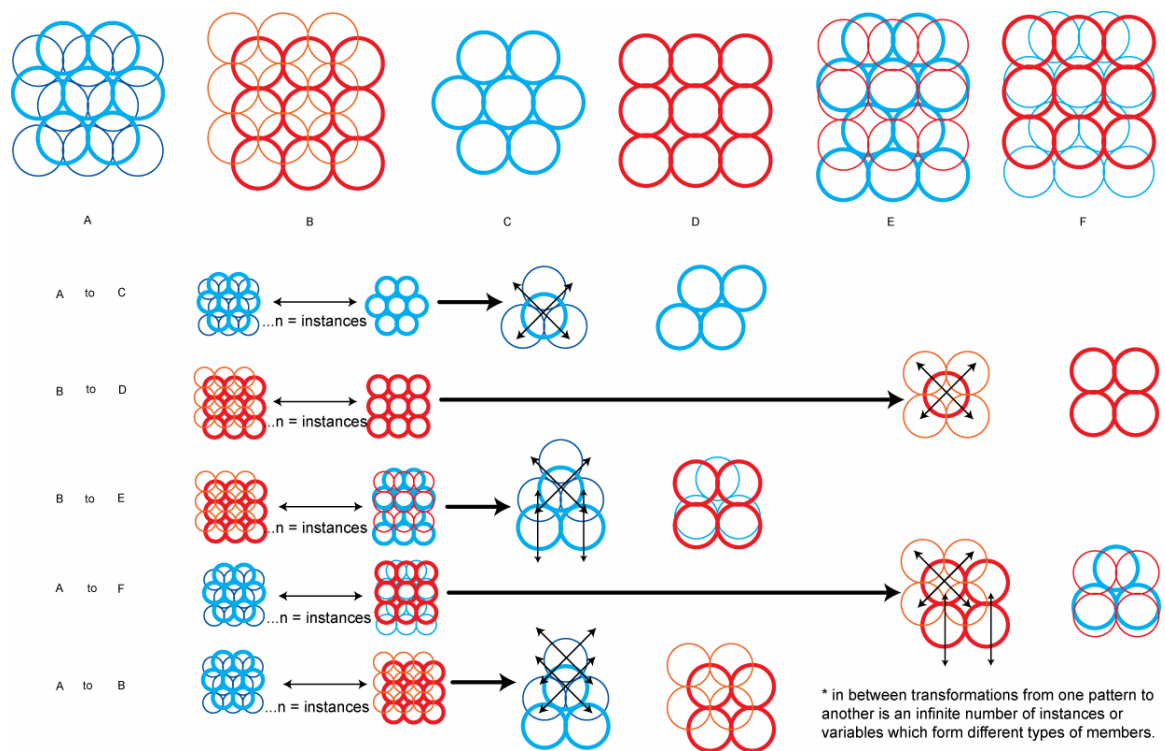


Figure 4-32: Matrix of Vertical Transformations

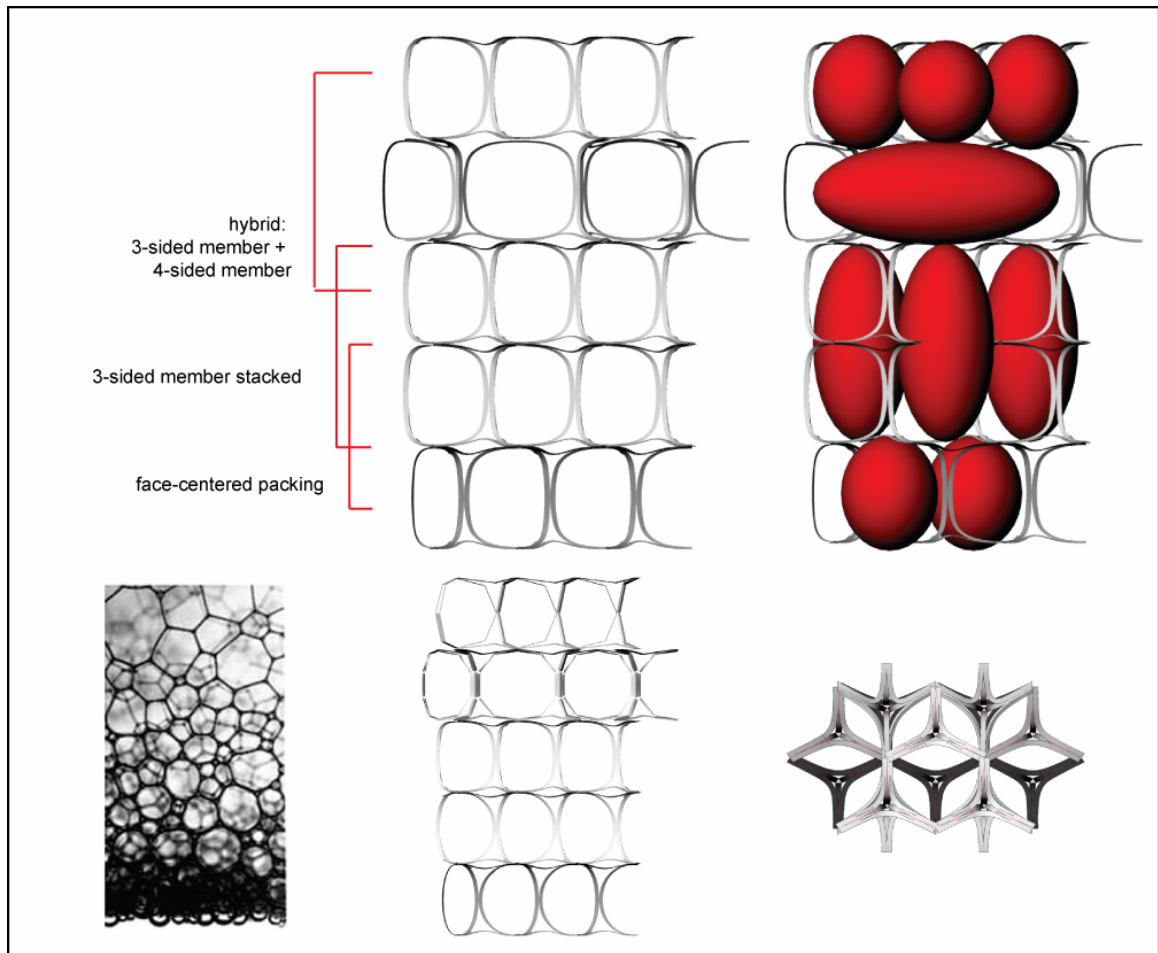


Figure 4-33: Example of Vertical Transformation of Members, Redrawn Considering Loads

Gravitational forces are an example of factors which come into play in the formation process of cellular aggregates. This vertical force creates size and weight transformations within the layers. Because of gravity, members should be more rounded at the bottom than the top because forces are greater at the bottom. Arches are more efficient in distributing loads. The figure above demonstrates this principle. The system itself is moving from a face-centered packing to a 3-sided member stack and a hybrid.

4.4.3 Horizontal Relationships

4.4.3.1 Face-Centered Packing – Hexagonal Packing – Hexagonal Dense Packing

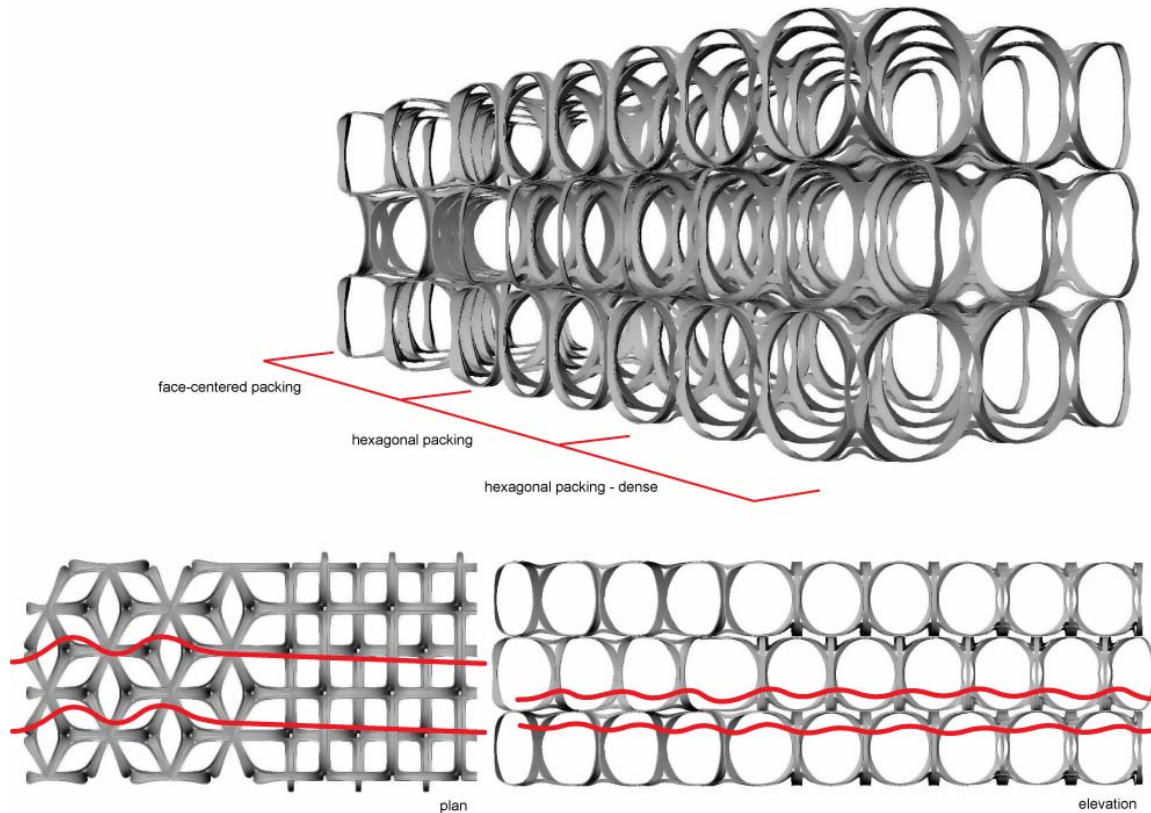


Figure 4-34: Horizontal Relationship _ Face-Centered Packing + Hexagonal Packing + Dense

These horizontal relationships demonstrate the movement from the different packing members. Connectivity in horizontal members are dependent on the number of branches (3-sided vs. 4-sided) within the proximity of the previous member. The results of this show a zigzag pattern horizontally (consistent with our understanding of packing systems). The plan view, on the other hand, shows an initial zigzag relationship which slowly straightens out; this is principally due to the horizontality of the square grid pattern.

4.4.3.2 Triangular Grid Stacked – Square Grid Stacked – Square Grid Dense Stacking

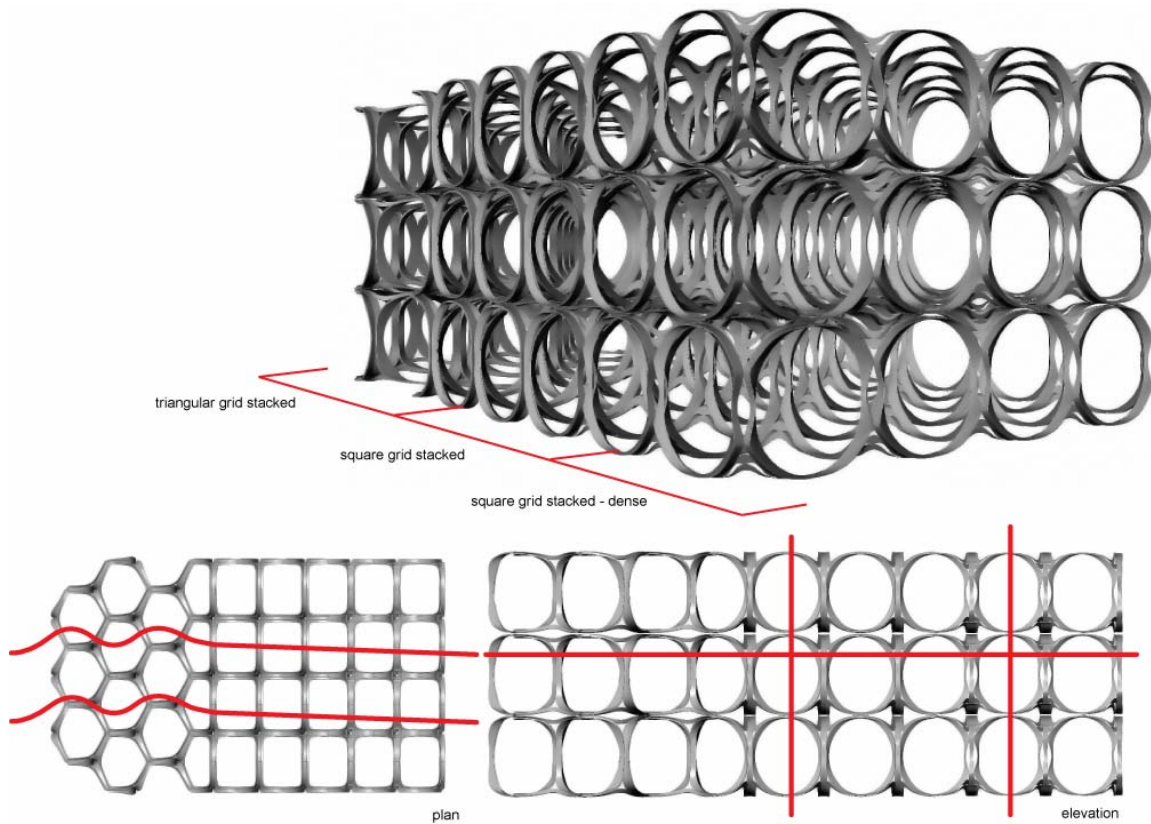


Figure 4-35: Horizontal Relationship _ Triangular Grid Stack + Square Grid Stack + Dense

This is similar to the previous model, but investigating horizontal relationships in stacked members. Predictably the results show a straight or direct relationship both vertically and horizontally. The plan view shows an initial zigzag pattern which eventually straightens out due to the square grid pattern.

4.4.3.3 Face-Centered Packing – Triangular Grid Stacked – Hybrid (3 sided member + 4-sided member)

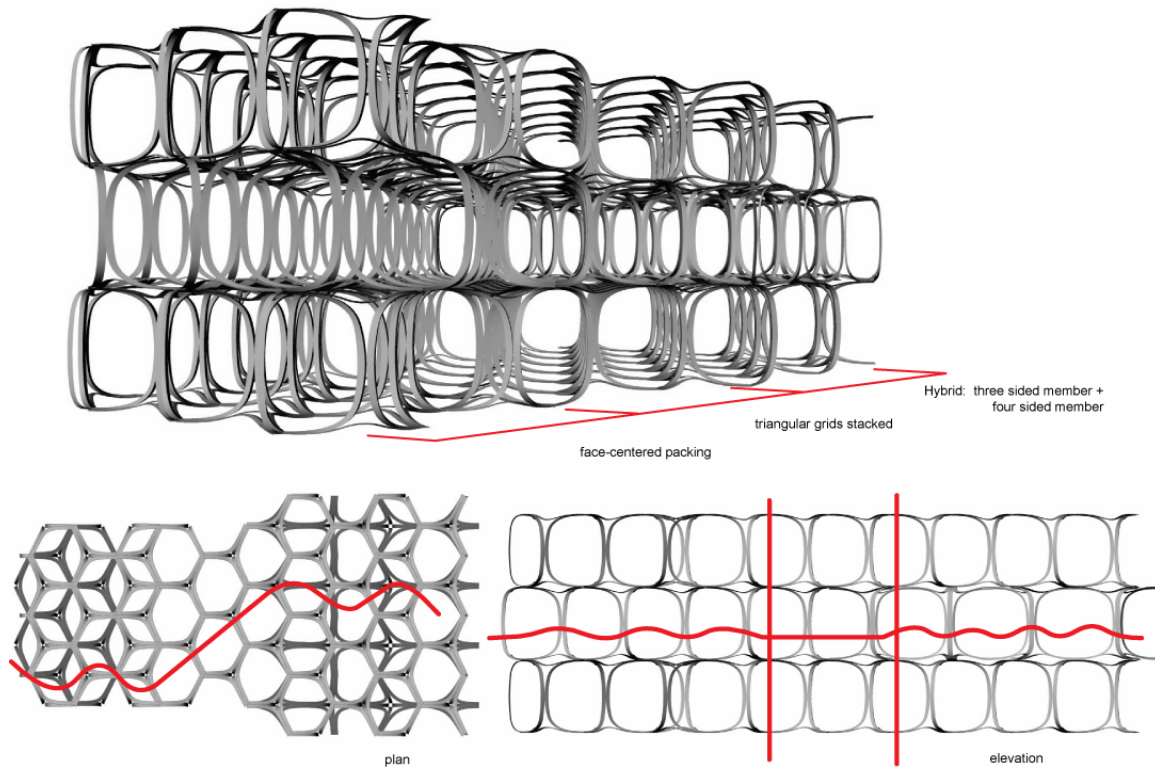


Figure 4-36: Horizontal Relationship _ Face-Centered Packing + Triangular Grid Stack + Hybrid

This next set of relationships goes from a packing to a stacking member, which in this case used a triangular grid or a 3-sided member, and finally into a hybrid of layers. The resultant horizontal relationship shows an almost continuous zigzag relationship with a break in the middle (straight line). This is primarily due to the stacking that occurs in the middle section. The plan view shows an interesting relationship, wherein the line starts out as zigzag patterns and in the middle becomes a straight diagonal line.

4.4.3.4 Hexagonal Packing – Hexagonal Dense Packing – Square Grid Stack – Hybrid (4-sided member + 3 sided member)

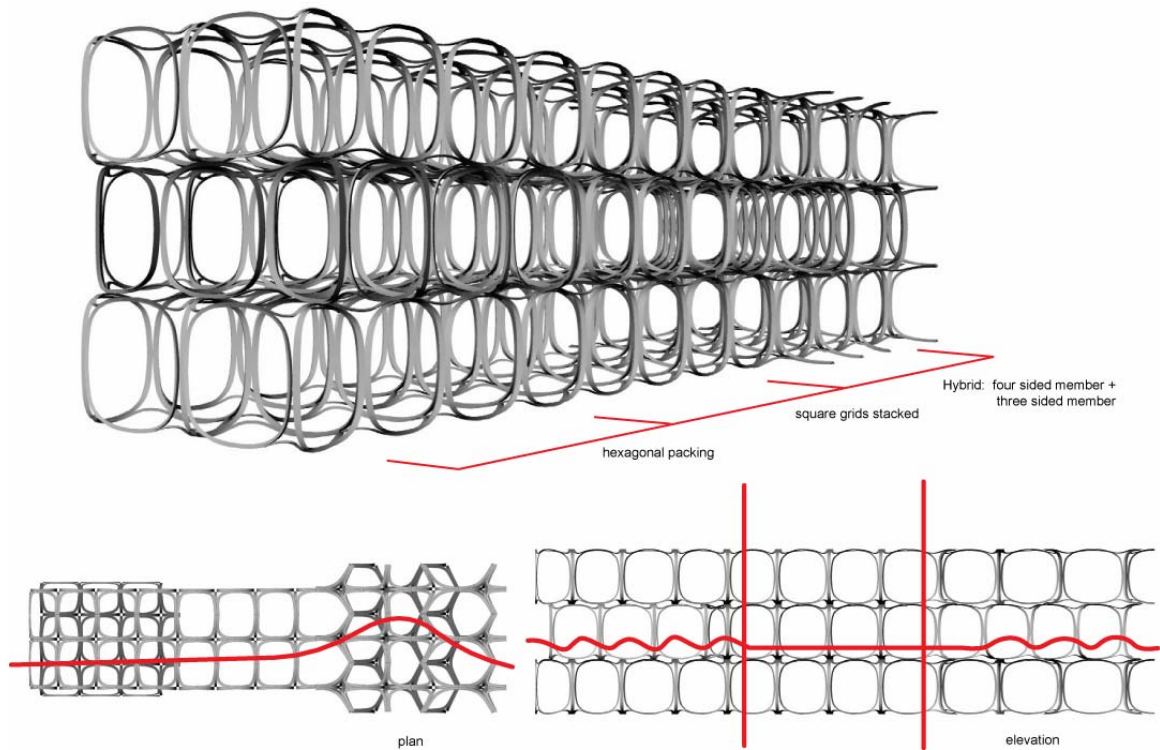


Figure 4-37: Horizontal Relationship _ Hexagonal Packing + Square Grid Stack + Hybrid

The last set of horizontal relationships goes from a packing to a stacking, using the 4-sided member, to a hybrid member. The major difference between this model and the previous one is that the plan view generates a straight line which morphs into a zigzag line in the end. The somewhat straight line is primarily due to the use of the square grid throughout the system.

4.4.4 Horizontal Transformations

Horizontal transformations are dependent on connectivity or tangency (i.e. the number of arms in a member which meets the succeeding member) for their potential to change. In the diagram below we see how three basic members may connect to form variability in spatial relationships (plan view). Although the diagram below shows the most basic connection which may occur, the members themselves may start to deform or distort due to connections in the vertical plane (as discussed in the section on vertical transformations). Hence, horizontal transformations are not only spatial relationships in planar view but also topological variations depending on vertical connections. In the comparison of an elevations with its plan (see previous section on horizontal relationships), we may find two varying spatial flows. The planar one respecting the connectivity of each member with each other while in elevation taking into consideration the stacking or packing method (stacking – linear pattern, packing – zigzag wave pattern).

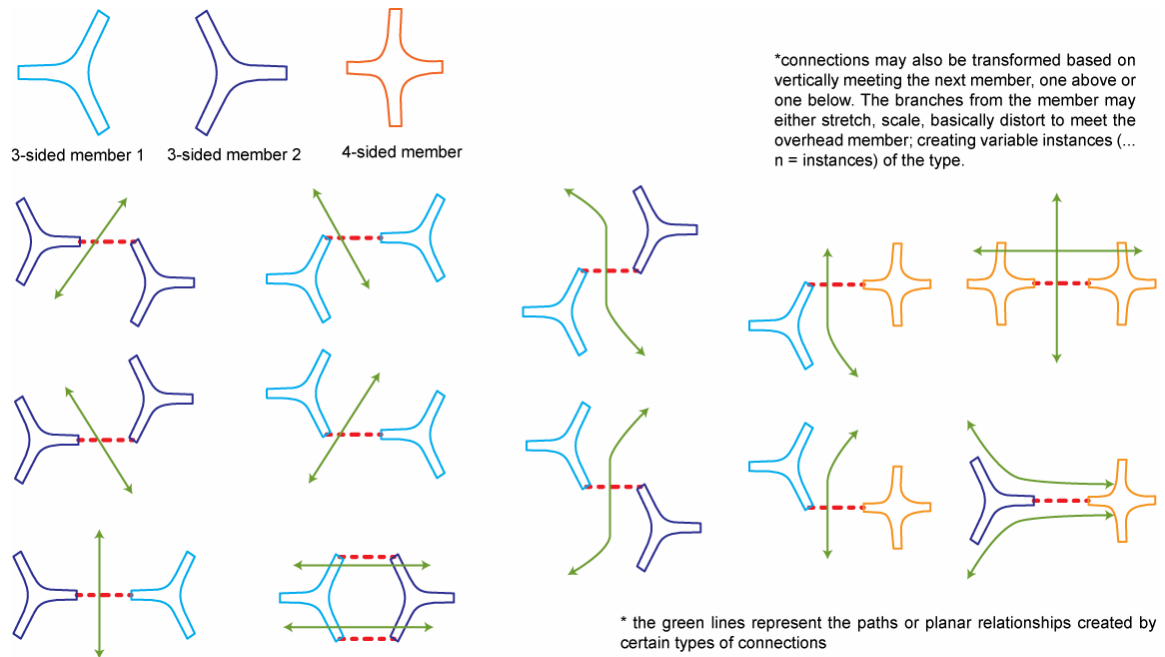


Figure 4-38: Matrix of Planar Connectivity Exhibiting Possible Horizontal Transformations

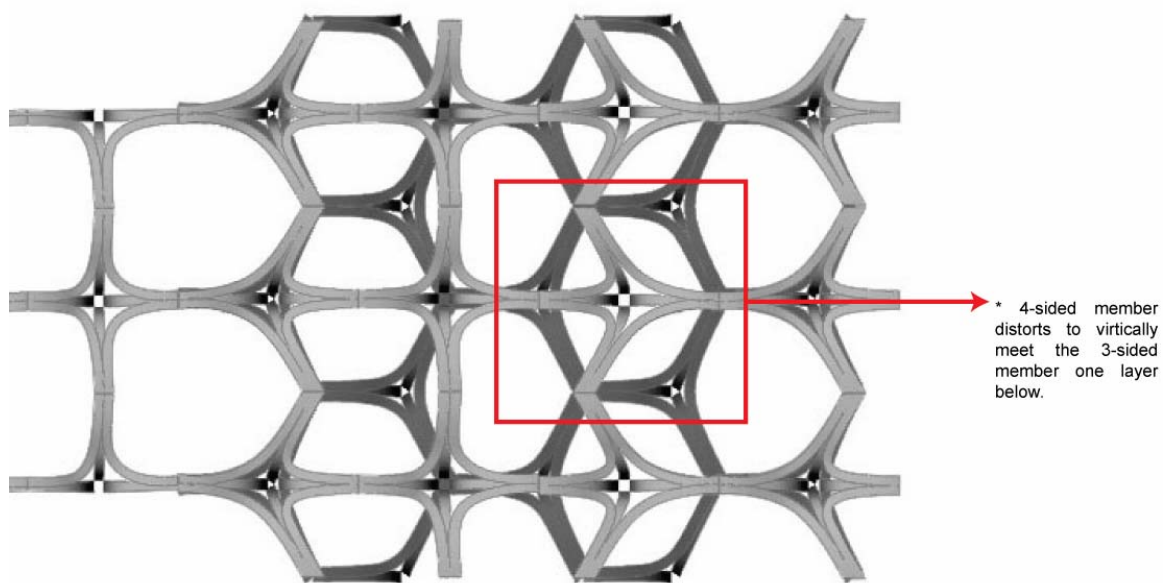


Figure 4-39: Detail of 4-Sided Member Distorted due to Vertical Relationship

The effect of the vertical relationship to the 4-Sided Member also exhibits a change in the horizontal planar configuration.

4.4.5 Conclusion

The digital processes of connectivity or tangency and patterning to create transformations clearly gives us a potential language of forms or members which can be used in the design of spaces. The efficacy of the research is its interconnectivity, meaning a space cannot be defined without affecting the structural member and certain structural members generate only certain spatial relationships.

This interconnectivity is a product of our research on natural systems, of bone, foam, soap bubbles. Just like in the formation of bones, the dense crystalline aggregates of members are a direct result of the spatial relationships of the cells that used to inhabit the voids. And similar to principles of surface tension and the creation of foam or soap bubbles, spatial relationships of voids (air pockets in foam) are dependent on external forces (vertical and horizontal relationships or forces) in their over all form-finding endeavor.

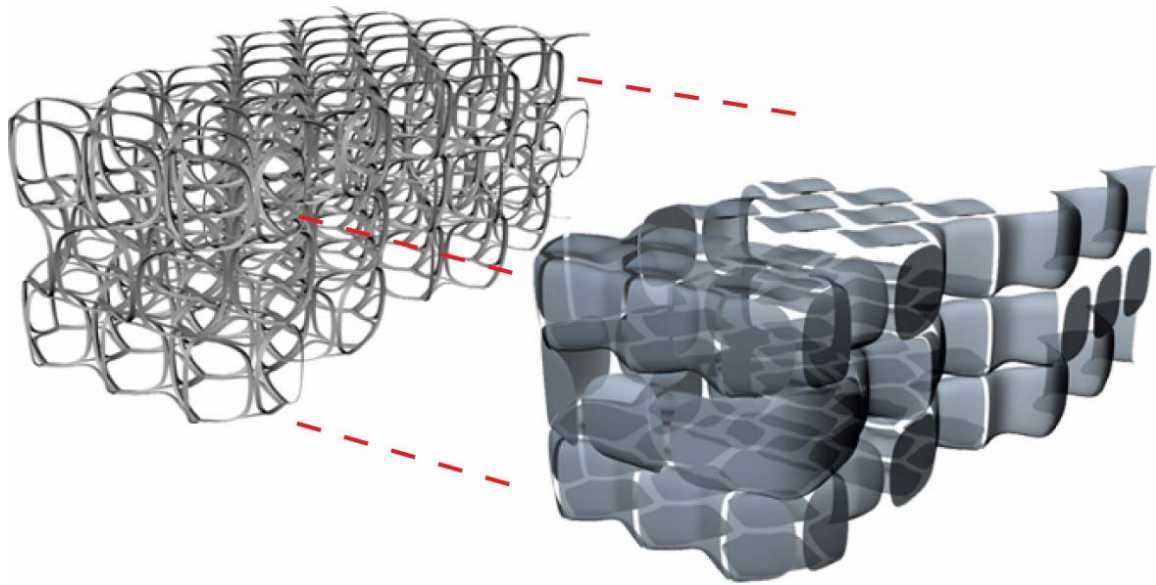


Figure 4-40: Axonometric View of Structure & Surface Fills



Figure 4-41: Rendering of Structure + Surface Fill System

REFERENCES

- Aranda, Benjamin, and Chris Lasch. Tooling. New York: Princeton Architectural Press. 2006.
- Balmond, Cecil. Informal. New York: Prestel. 2002.
- Bonabeau, Eric, Marco Dorigo, and Guy Theraulaz. Swarm Intelligence: From Natural to Artificial Systems. New York: Oxford University Press. 1999.
- Burry, Mark. Expiatory Church of the Sagrada Família: Antoni Gaudí. London: Phaidon Press. 1993.
- Conway, J.H., and N.J.A. Sloane. Sphere Packings, Lattices, and Groups. New York: Springer. 1999.
- Critchlow, Keith. Order in Space. New York: Thames & Hudson. 1969.
- Devlin, Keith. Mathematics, the Science of Patterns: The Search for Order in Life, Mind, and the Universe. New York: Scientific American Library. 1994.
- Eduardo, Juan. The Genesis of Gaudian Architecture. New York: G. Wittenborn. 1967.
- Emerton, Norma. The Scientific Reinterpretation of Form. Ithaca, N.Y.: Cornell University Press. 1984.
- Fuller, R. Buckminster. Your Private Sky: Discourse. Baden, Switzerland: Lars Muller. 2001.
- Hensel, Michael, and Achim Mente. "Material and Digital Design Synthesis: Integrating Material Self-Organisation, Digital Morphogenesis, Associative Parametric Modeling and Computer-Aided Manufacturing". Architectural Design. 76.2 (2006): 88-95.
- Hensel, Michael. "Towards Self-Organisational and Multiple-Performance Capacity in Architecture". Architectural Design. 76.2 (2006): 5-17.
- Hoberman, Chuck. "Unfolding Architecture". Architectural Design. 63.3-4 (1993): 56-59
- Holland, John H. Emergence: From Chaos to Order. Reading, Mass.: Addison-Wesley. 1998.
- Johnson, Steven. Emergence: The Connected Lives of Ants, Brains, Cities, and Software. New York: Scribner. 2001.
- Kolarevic, Branko, ed. Architecture in the Digital Age: Design and Manufacturing. New York: Spon Press. 2003.
- Leach, Neil, David Turnbull, and Chris Williams. Digital Tectonics. Chichester: Wiley-Academy. 2004.

Martin, R. Bruce, and David B. Burr. Structure, Function, and Adaptation of Compact Bone. New York: Raven Press. 1989.

Otto, Frei, and Bodo Rasch. Finding Form: Towards an Architecture of the Minimal. Germany: Axel Menges. 1995.

Otto, Frei. Tensile Structures. Cambridge, Mass.: M. I. T. Press. 1967-69.

Otto, Frei. Complete Works: Lightweight Construction, Natural Design. Boston: Birkhauser. 2005.

Philip, Drew. Frei Otto: Form and Structure. Boulder, Colorado: Westview Press. 1976.

Ramírez, Juan Antonio. The Beehive Metaphor: From Gaudí to Le Corbusier. London: Reaktion. 2000.

Rice, Peter. An Engineer Imagines. London: Artemis. 1996.

Speaks, Michael. "Design Intelligence: Or Thinking After the End of Metaphysics". Architectural Design. 72.5 (2002): 4-6.

Spuybroek, Lars. Nox: Machining Architecture. New York: Thames and Hudson. 2004.

Tehrani, Nader and Monica Ponce de Leon. Office dA. Gloucester, Mass.: Rockport Publishers. 2000.

Testa, Peter, and Devyn Weiser. "Emergent Structural Morphology". Architectural Design. 72.1 (2002): 13-16.

Thompson, D'Arcy W. On Growth & Form. Cambridge: Cambridge University Press. 1961.

Weaire, Denis. The Kelvin Problem: Foam Structures of Minimal Surface Area. London; Bristol, PA: Taylor & Francis. 1996.

Weinstock, Michael. "Morphogenesis & the Mathematics of Emergence". Architectural Design. 74.3 (2004): 10-17.

Weinstock, Michael. "Emergence in Architecture: the Emergence and Design Group". Architectural Design. 74.3 (2004): 6-9.

Weinstock, Michael, Achim Menges, Michael Hensel. "Fit fabric: versatility through redundancy and differentiation". Architectural Design. 74.3 (2004): 40-47.

Wilson, A.J., ed. Foams: Physics, Chemistry, and Structure. London; New York: Springer-Verlag. 1989.

<http://ciks.cbt.nist.gov/~garbocz/closedcell/node5.html>, 1995

<http://www.tiem.utk.edu/~gross/bioed/webmodules/spherepacking.htm>, 2000

http://training.seer.cancer.gov/module_anatomy/unit3_3_bone_growth.html, 2000

http://science.nasa.gov/headlines/y2003/09jun_foam.htm, June 9, 2003

http://www.materialsystems.org/Projects/20041001_manifold/full/manifold.html, 2004

<http://www.polymerexpert.biz/PolymersandComposites.html>, 2004

http://en.wikipedia.org/wiki/Plateau%27s_laws, January 20, 2007